Chinese Journal of Aeronautics, (2017), **xxx**(xx): xxx–xxx

Chinese Society of Aeronautics and Astronautics
& Beihang University

Chinese Journal of Aeronautics

cja@buaa.edu.cn
www.sciencedirect.com

A multi-order method for predicting stability of a multi-delay milling system considering helix angle and run-out effects

Shanglei JIANG, Yuwen SUN *

Key Laboratory for Precision and Non-Traditional Machining Technology of the Ministry of Education, Dalian University of Technology, Dalian 116024, China

Received 27 February 2017; revised 11 May 2017; accepted 20 May 2017

KEYWORDS

Cutting force;
Floquet theory;
Milling stability;
Multi-delay milling system;
Run-out

Abstract In this paper, a multi-delay milling system considering helix angle and run-out effects is firstly established. An exponential cutting force model is used to model the interaction between a work-piece and a cutting tool, and a new approach is presented for accurately calibrating exponential cutting force coefficients and cutter run-out parameters. Furthermore, based on an implicit multi-step Adams formula and an improved precise time-integration algorithm, a novel stability prediction method is proposed to predict the stability of the system. The involved time delay term and periodic coefficient term are integrated as a comprehensive state term in the integral response which is approximated by the Adams formula. Then, a Floquet transition matrix with an arbitrary-order form is constructed by using a series of matrix multiplication, and the stability of the system is determined by the Floquet theory. Compared to classical semi-discretization methods and full-discretization methods, the developed method shows a good performance in convergence, efficiency, accuracy, and multi-order complexity. A series of cutting tests is further carried out to validate the practicability and effectiveness of the proposed method. The results show that the calibration process needs a time of less than 5 min, and the stability prediction method is effective.

1. Introduction

Milling operations are widely used in the aerospace industry for machining various highly expensive components, such as aero-engine blisks, impellers, blades, casings, and so on. These components are mostly made of aluminum alloys, or difficult-to-cut titanium and nickel alloys. In the milling process, chatter is an undesirable phenomenon that inevitably deteriorates workpiece quality and even causes damages to CNC machine tools.[1] How to avoid chatter is a key issue to

---

* Corresponding author.
E-mail address: xiands@dlut.edu.cn (Y. SUN).

Peer review under responsibility of Editorial Committee of CJA.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.cja.2017.08.005

Please cite this article in press as: JIANG S, SUN Y A multi-order method for predicting stability of a multi-delay milling system considering helix angle and run-out effects, *Chin J Aeronaut* (2017), http://dx.doi.org/10.1016/j.cja.2017.08.005

ensure a stable cut with a high material removal rate. Chatter stability prediction plays an important role in selecting machining parameters in order to achieve a chatter-free operation. Generally, it utilizes stability lobes to classify machining parameter combinations into stable and unstable regions in a diagram,[2] namely the stability lobe diagram (SLD). An optimal cutting parameter combination chosen in a stable region not only avoids chatter, but also improves machining productivity. Therefore, it is particularly necessary and important to seek an effective prediction method.

Apart from the trial and error method, some feasible methods including analytical and numerical methods have been proposed in the frequency domain and the discrete time domain. Altintas and Budak[3] proposed the first analytical solution in the frequency domain. Their method, known as the single-frequency solution (SFS), can give a rapid and accurate computation of the SLD in a large radial immersion case. In order to improve the prediction accuracy in a small radial immersion case, a multi-frequency solution (MFS) was further proposed by Budak and Altintas[4], which uses the higher harmonics of directional factors instead of the average ones used in the SFS. Alternately, based on the Floquet theory of delay-differential equations (DDEs), modeling in the discrete time domain is also a good choice to achieve accurate stability predictions. Bayly et al.[5] solved discrete time equations by using temporal finite elements analysis to determine stability boundaries. Butcher et al.[6] proposed the Chebyshev collocation method in the discrete time domain to predict the stability of a time-periodic DDE. Both methods use one matrix to construct the Floquet transition matrix (FTM) in a similar way, and they are very competitive for their rates of convergence. However, they are only easy to use for single-delay cases and not quite suitable for cases with varying or multiple time delays.

On the other hand, Insperger and Stépán[7] investigated the periodic motion of time-period DDEs using the semi-discretization (SD) method, in which only the delay term and the periodic coefficient term are discretized. An updated version of the SD method,[8] known as the zeroth-order SD (0th SD) method, was then applied to predict the stability of milling processes, in which the delay term was discretized as a weighted sum of two neighboring discrete state values. Furthermore, a first-order SD (1st SD) method[9] was developed to essentially increase the efficiency of the original SD method, in which the delay term was then approximated by linear interpolation of two neighboring discrete state values. Alternatively, Ding et al.[10] proposed a first-order full-discretization (1st FD) method with a faster computational efficiency. In addition to discretizing the delay term and the periodic coefficient term, a portion of the actual time-domain state term is discretized in the FD method as well. Both SD and FD methods use a series of matrix multiplication to construct the FTM in another similar way, and they can be extended to predict the stability of a milling system with varying or multiple time delays. Insperger[9] compared SD with FD methods in a same scheme and proved that the 1st FD method converges slower than the same-order SD method. Later, second-order FD (2nd FD)[11] and high-order FD methods[12–14] were further developed to improve the convergence rate of FD methods. However, the construction of the FTM becomes more complicated, especially for multi- or varying-delay systems. Recently, Ding et al.[15] developed an efficient numerical integration (NI) method and Zhang et al.[16] proposed a compact Simpson method for the stability analysis of milling processes, respectively based on an integration scheme and a differential scheme. It was found that these methods are also analogous to temporal finite elements analysis and the Chebyshev collocation method in constructing the FTM. Besides, Zhou et al.[17] predicted the stability in end milling of aero-engine casings using an analytical method. Luo et al.[18] presented a new time-domain model of mechanics and dynamics of the cutter exit process.

This paper proposes an efficient, accurate, and compact stability prediction for a multi-delay milling system. The delay term and the periodic coefficient term are integrated as a comprehensive state term in the integral response of time-period DDEs which is approximated by a multi-step implicit Adams formula, and the time-domain state term is not discretized. A compact and arbitrary-order FTM is constructed by using a series of matrix multiplication. An improved precise time-integration algorithm is used to calculate the resulting exponential matrices rapidly. Furthermore, considering that different cutting force models and corresponding calibration accuracies of cutting force coefficients significantly affect the reliability of stability lobes[19,20] but the cutting force models in most of the above works are linear, an exponential force model is employed and a new approach is also presented to accurately calibrate exponential cutting force coefficients (ECFCs) and cutter run-out parameters (CRPs) simultaneously.

2. Modeling of the milling dynamics

The milling cutter is modeled as a mass-spring-damper system with two degrees of freedom (2-DOFs) respectively in the X direction (parallel to the tool feed) and the Y direction (perpendicular to the tool feed). It is assumed to be flexible as opposed to the rigid workpiece as shown in Fig. 1. $O_{\mathrm{G}}(t)$ is the geometric center of the milling cutter at the current cutting instant t. $O_{\mathrm{G}}(t-\tau)$ is the geometric center of the milling cutter at the previous cutting instant $t-\tau$, where τ is the time delay. X_{R} and Y_{R} are the orthogonal coordinate axes with their coordinate origins lying at $O_{\mathrm{G}}(t)$. $j-1$, j, and $j+1$ represent the previous, current, and next cutter teeth, respectively. k_x and

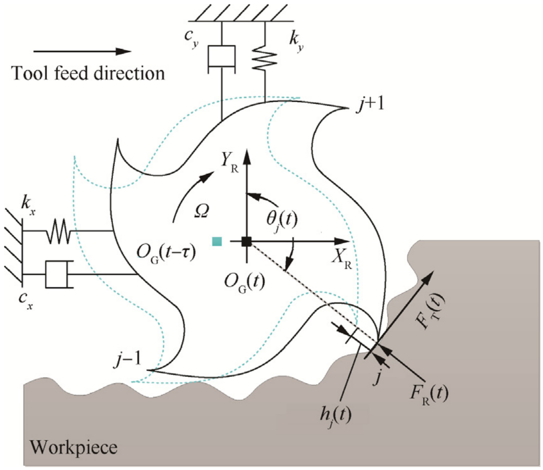

Fig. 1 Schematic of a 2-DOF milling process.

Please cite this article in press as: JIANG S, SUN Y A multi-order method for predicting stability of a multi-delay milling system considering helix angle and run-out effects, *Chin J Aeronaut* (2017), http://dx.doi.org/10.1016/j.cja.2017.08.005