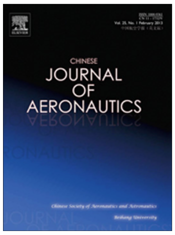
Chinese Journal of Aeronautics, (2018), **xxx**(xx): xxx–xxx

Chinese Society of Aeronautics and Astronautics
& Beihang University

Chinese Journal of Aeronautics

cja@buaa.edu.cn
www.sciencedirect.com



Sparse grid-based polynomial chaos expansion for aerodynamics of an airfoil with uncertainties

Xiaojing WU, Weiwei ZHANG *, **Shufang SONG, Zhengyin YE**

School of Aeronautics, Northwestern Polytechnical University, Xi'an 710072, China

Received 6 March 2017; revised 2 August 2017; accepted 20 August 2017

KEYWORDS

Non-intrusive polynomial chaos;
Sparse grid;
Stochastic aerodynamic analysis;
Uncertainty sensitivity analysis;
Uncertainty quantification

Abstract The uncertainties can generate fluctuations with aerodynamic characteristics. Uncertainty Quantification (UQ) is applied to compute its impact on the aerodynamic characteristics. In addition, the contribution of each uncertainty to aerodynamic characteristics should be computed by uncertainty sensitivity analysis. Non-Intrusive Polynomial Chaos (NIPC) has been successfully applied to uncertainty quantification and uncertainty sensitivity analysis. However, the non-intrusive polynomial chaos method becomes inefficient as the number of random variables adopted to describe uncertainties increases. This deficiency becomes significant in stochastic aerodynamic analysis considering the geometric uncertainty because the description of geometric uncertainty generally needs many parameters. To solve the deficiency, a Sparse Grid-based Polynomial Chaos (SGPC) expansion is used to do uncertainty quantification and sensitivity analysis for stochastic aerodynamic analysis considering geometric and operational uncertainties. It is proved that the method is more efficient than non-intrusive polynomial chaos and Monte Carlo Simulation (MSC) method for the stochastic aerodynamic analysis. By uncertainty quantification, it can be learnt that the flow characteristics of shock wave and boundary layer separation are sensitive to the geometric uncertainty in transonic region. The uncertainty sensitivity analysis reveals the individual and coupled effects among the uncertainty parameters.

© 2018 Chinese Society of Aeronautics and Astronautics. Production and hosting by Elsevier Ltd. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

1. Introduction

A vast amount of uncertainties exist in the practical aircraft design and application, which can cause fluctuations of aircraft performance. Therefore, it is important to take these uncertainties into account at the beginning of aircraft design.[1,2] Recently, many researches have concerned the topics.[3–8] The Uncertainty Quantification (UQ) and uncertainty sensitivity analysis of aerodynamics are concerned in the paper.

Computational Fluid Dynamics (CFD) technology has been widely used to solve problems in fluid mechanics with the development of computer technology. Traditional CFD simulation is deterministic. However, a variety of uncertainties are inevitably introduced into CFD simulation due to the increasing complexity of the problems. This leads to the mismatch between the results of CFD simulation and the actual

---

* Corresponding author.
E-mail address: aeroelastic@nwpu.edu.cn (W. ZHANG).

Peer review under responsibility of Editorial Committee of CJA.

Production and hosting by Elsevier

https://doi.org/10.1016/j.cja.2018.03.011
1000-9361 © 2018 Chinese Society of Aeronautics and Astronautics. Production and hosting by Elsevier Ltd.

This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

Please cite this article in press as: WU X et al. Sparse grid-based polynomial chaos expansion for aerodynamics of an airfoil with uncertainties, *Chin J Aeronaut* (2018), https://doi.org/10.1016/j.cja.2018.03.011

results. The UQ in CFD simulation has gained extensive attention in Ref.[9]. Sources and classifications of uncertainty in CFD were described in Ref.[10]. Several UQ strategies have been used in CFD, including Monte Carlo Simulation (MCS) method, moment method and Polynomial Chaos (PC) in Ref.[11]. MCS is a statistical method, which needs many samples to accurately quantify uncertainty. Moment methods are suitable to solve the problem of small parameter uncertainty or linear model. Recently, PC which is based on the spectral representation of the uncertainty has been adopted in UQ for fluid problems in Refs.[12,13]. PC methods can be divided into intrusive and non-intrusive ones according to the coupling ways with CFD solvers. In general, an intrusive approach is adopted to obtain unknown polynomial coefficients by projecting resulting equations into basis functions for different modes, and it requires the modification of CFD codes, which may be difficult and time-consuming for complex problems such as Navier–Stokes equations. To overcome the shortcomings of intrusive polynomial chaos, Non-Intrusive Polynomial Chaos (NIPC) has been developed. The CFD is regarded as a black box model without changing the CFD program codes in non-intrusive methods. There are two different sampling approaches to build NIPC: random sampling and deterministic sampling. Random sampling methods use MCS to evaluate the unknown coefficients, but their convergence rate is low. Deterministic sampling methods use the quadrature to evaluate the unknown coefficients. The quadrature-based methods are more efficient than random sampling methods for low-dimensional problems. However, they are inefficient for relatively high-dimensional problems because of the exponential rising of quadrature points with the increasing dimensions. The UQ based on PC and its applications in fluid mechanics were comprehensively reviewed in Refs.[14,15].

Recently, the NIPC is sufficiently used for stochastic aerodynamic analysis with operational uncertainties. A subsonic aerodynamic analysis was conducted on a NACA0012 airfoil with an uncertain free stream velocity using a commercial flow solver in Ref.[16]. They proved that an uncertain free stream velocity led to the highest variation in pressure on the upper surface near the leading edge. Transonic stochastic response of two-dimensional airfoil to parameter uncertainty (Mach number Ma and angle of attack α) is focused using generalized Polynomial Chaos (gPC) in Ref.[8]. Two kinds of non-linearities are critical to transonic aerodynamics in their study: the shock characteristics and boundary-layer separation. A stochastic investigation of flows about NACA0012 airfoil was conducted at transonic speeds in Ref.[17]. A point-collocation NIPC method was used to quantify uncertainty of aerodynamic characteristics with uncertain variables Ma and α in the transonic-wing case in Ref.[18]. A stochastic fluid analysis on 3D wind blades considering the wind speed as an uncertain parameter was conducted in Ref.[19]. In their studies, when the flow separation appears, the separation vortex region corresponds to the maximum variation area which extends to the trailing edge and even to the whole suction side. However, the stochastic aerodynamic analysis considering geometric uncertainty was rarely involved. The geometric uncertainty on an aerodynamic surface resulting from manufacturing errors has significant effect on the aerodynamic performance. It is impractical to remove the impact of these geometric variations by improving the manufacturing tolerance due to the high cost of the precise surface manufacturing technique. In other words, the geometric uncertainty due to manufacturing errors is unavoidable. Therefore, it is important and necessary to conduct a stochastic aerodynamic analysis considering geometric uncertainty. Nevertheless, the description of the geometric uncertainty is difficult in a computing environment and many random variables are needed to represent aerodynamic surface fluctuations. Therefore, the quadrature-based NIPC method is inefficient in solving this high-dimensional stochastic problem.

To improve the computational efficiency of traditional quadrature-based NIPC method for high-dimensional problems, the sparse grid numerical integration is introduced to solve the coefficients of PC. The sparse grid technique extends one-dimensional formulae to higher dimensions by tensor product and then selects sampling points according to Smolyak theory in Ref.[20]. It has been widely used in numerical integration and interpolation[21,22] as well as data mining.[23] Recently, a new sparse grid-based method has been developed for UQ in Ref.[24]. From their research, it can be known that when the dimension of the uncertain variables is larger than 5, the computational cost required by the sparse grid method is much smaller than that required by tensor product method. A sparse grid interpolant was developed to solve the high-dimensional stochastic partial different equations in Ref.[25]. Sparse grid collocation schemes were applied to stochastic natural convection problems in Ref.[26]. Sparse grids-based stochastic approximations with applications to subsonic steady flow about a NACA0015 airfoil in the presence of geometrical and operational uncertainties with both simplified aerodynamics model and Reynolds-Averaged Navier–Stokes (RANS) simulation was presented in Ref.[27].

In this paper, a Sparse Grid-based Polynomial Chaos (SGPC) method is constructed to UQ and sensitivity analysis for transonic aerodynamics considering airfoil geometric and operational uncertainties in detail. The paper is structured as follows: Section 2 introduces the mathematical formulations of the sparse grid technique; in Section 3, the SGPC is built; in Section 4, a stochastic aerodynamic analysis considering geometric and operational uncertainties is conducted in detail; Section 5 outlines several useful conclusions.

2. Sparse grid numerical integration

Sparse grid technique selects sampling points under Smolyak theory, which uses a weighted linear combination of special tensor products to reduce the grid size. The sparse grid has been successfully used for numerical integration. Locations and weights of the univariate quadrature points with a range of accuracy are determined in each dimension, and then the univariate quadrature point sets are extended to form a multi-dimensional grid using the sparse grid theory. The introduction of Sparse Grid Numerical Integration (SGNI) is as follows:

For the one-dimensional integral $\int_{\Omega_i} f(x)\varphi_i(x)\mathrm{d}x$, the quadrature formula is defined as

Please cite this article in press as: WU X et al. Sparse grid-based polynomial chaos expansion for aerodynamics of an airfoil with uncertainties, *Chin J Aeronaut* (2018), https://doi.org/10.1016/j.cja.2018.03.011

Download English Version:

https://daneshyari.com/en/article/7153648

Download Persian Version:

https://daneshyari.com/article/7153648

Daneshyari.com