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Interface flux reconstruction method based on optimized weight essentially non-oscillatory scheme

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Multi-size mesh

Abstract Aimed at the computational aeroacoustics multi-scale problem of complex configurations discretized with multi-size mesh, the flux reconstruction method based on modified Weight Essentially Non-Oscillatory (WENO) scheme is proposed at the interfaces of multi-block grids. With the idea of Dispersion-Relation-Preserving (DRP) scheme, different weight coefficients are obtained by optimization, so that it is in WENO schemes with various characteristics of dispersion and dissipation. On the basis, hybrid flux vector splitting method is utilized to intelligently judge the amplitude of the gap between grid interfaces. After the simulation and analysis of 1D convection equation with different initial conditions, modified WENO scheme is proved to be able to independently distinguish the gap amplitude and generate corresponding dissipation according to the grid resolution. Using the idea of flux reconstruction at grid interfaces, modified WENO scheme with increasing dissipation is applied at grid points, while DRP scheme with low dispersion and dissipation is applied at the inner part of grids. Moreover, Gauss impulse spread and periodic point sound source flow among three cylinders with multi-scale grids are carried out. The results show that the flux reconstruction method at grid interfaces is capable of dealing with Computational AeroAcoustics (CAA) multi-scale problems.

1. Introduction

In the past few decades, high order finite difference methods have been developed in Computational AeroAcoustics (CAA). Most of these methods evolve from the Dispersion-Relation-Preserving (DRP) scheme proposed by Tam and Webb,[1] or the compact finite difference scheme by Lele.[2] Though having gained great success in CAA, the high order finite difference methods cannot be applied to practical CAA multi-scale problems with complex geometry difficultly, since

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it requires that the flux across the grid interface has to be continuous. When facing aeroacoustics problems with complex geometries, multi-size structured grids are usually employed, which greatly improves the capability of application in some practical engineering problems such as aircraft high lift systems and landing gears. However, how to construct the interface flux efficiently and stably is still a hot issue.

Aimed at multi-scale CAA problems, Tam and Kurbatskii[3] proposed low-dissipation & low-dispersion DRP scheme by optimizing the interpolation parameters with comparing multiple grid scales ratio. Besides, Tam improved the stencil DRP scheme and artificial selective damping terms which are used in the interface flux construction, in order to compute the flux accurately.

WENO scheme was initially proposed by Liu and Osher[4] based on Essentially Non-Oscillatory (ENO) scheme.[5] The weights depend on the local smoothness of the data. Smoothness measurements cause stencils that span large flow field gradients to have relative small weights; any candidate stencil containing a shock receives a nearly zero weight. In completely smooth regions, weights revert to optimal values, where optimal value is defined by maximum order of accuracy or maximum bandwidth. In the following years, Jiang and Shu[6,7] cast WENO scheme into finite difference form. This scheme, which is referred to as WENO-JS hereinafter, can be capable of resolving shocks with high resolution. However, WENO-JS is too dissipative for the detailed simulation of turbulent flow. Martin et al.[8,9] developed two new formulations of a symmetric WENO method for the direct numerical simulation of compressible turbulence. The schemes are designed to maximize order of accuracy and bandwidth, while minimizing dissipation. The formulations and the corresponding coefficients are introduced. Numerical solutions to canonical flow problems are used to determine the dissipation and bandwidth properties of the numerical schemes. In addition, the suitability and accuracy of the bandwidth-optimized schemes for direct numerical simulations of turbulent flows are assessed in decaying isotropic turbulence and supersonic turbulent boundary layers. Wu et al.[10] presented a maximum order preserving optimized WENO scheme which is a weighted average of the maximum order scheme and an optimized scheme. Hou et al.[11] modified the weights of WENO scheme to be smoother, which can eliminate the fluctuation within the grid variation. Lin and Hu[12] presented two groups of WENO schemes based on the dissipation and dissipation, which are investigated for computational aeroacoustics.

In this paper, the modified WENO scheme is deduced firstly. Then, we apply modified WENO scheme to establish the model of interface flux. On this basis, the accuracy of the model in multi-scale grid problems has been verified by several standard verification cases. The numerical simulation results with interface flux method are presented in Section 5, and the brief conclusions are given in Section 6.

2. Optimized WENO scheme

In the design of a traditional WENO scheme, the practice is to maximize the order of accuracy of the WENO scheme given the size of the difference stencil. However, high order schemes may not be the best for CAA problems. Aimed at short waves, the finite difference scheme needs to maintain its low-dissipation and low-dispersion property, as shown by Tam and Webb. To fix the idea, the initial value problem associated with the scalar wave equation is considered as follows:

$$\frac{\partial u(x,t)}{\partial t} + a\frac{\partial u(u,t)}{\partial x} = 0 \quad \text{with } u(x,0) = u_0(x) \tag{1}$$

where x is coordinate, t is time, u is function of x and t, a is constant number.

Given a uniform grid $x_i = i\Delta x$ with the same grid spacing Δx, the semi-discretized form of Eq. (1) is

$$\frac{\partial u_i}{\partial t} + a\frac{1}{\Delta x}(u_{i+1/2} - u_{i-1/2}) = 0 \tag{2}$$

where i is integer number, $au_{i+1/2}$ and $au_{i-1/2}$ are numerical fluxes which depend on k ($r+s+1=k$, $r \geqslant 0$, $s \geqslant 0$; r and s are different integer numbers respectively) grid points including x_i itself, i.e.,

$$u_{i+1/2} = \sum_{j=0}^{k-1} c_{rj}u_{i-r+j} \tag{3}$$

Here c_{rj} could be obtained by achieving k-th order accuracy in Taylor series truncation expansion. Considering five-point-stencil WENO-JS scheme, according to Ref.[6], it would be divided to 3 candidate stencils $\{S_1, S_2, S_3\}$. Each of candidate stencils has three nodes, as shown in Fig. 1. The second order polynomial approximation $u^k(x) = a_kx^2 + b_kx + c_k$ could be obtained by using the function value of $u(x)$ in each candidate stencil, where $k = 1, 2, 3$.

In each candidate stencil, the formula for $u^k(x)$ could be obtained by Taylor series expansion at x_j as follows:

$$\begin{cases} u^0(x) = \dfrac{u_{j-2} - 2u_{j-1} + u_j}{2\Delta x^2}x^2 + \dfrac{u_{j-2} - 4u_{j-1} + 3u_j}{2\Delta x}x + \dfrac{-u_{j-2} + 2u_{j-1} + 23u_j}{24} \\ u^1(x) = \dfrac{u_{j-1} - 2u_j + u_{j+1}}{2\Delta x^2}x^2 + \dfrac{u_{j+1} - u_{j-1}}{2\Delta x}x + \dfrac{-u_{j-1} + 26u_{j-1} - u_{j+1}}{24} \\ u^2(x) = \dfrac{u_j - 2u_{j+1} + u_{j+2}}{2\Delta x^2}x^2 + \dfrac{-3u_j + 4u_{j+1} - u_{j+2}}{2\Delta x}x + \dfrac{23u_j + 2u_{j+1} - u_{j+2}}{24} \end{cases} \tag{4}$$

When $x = \Delta x/2$, Eq. (4) can be approximated by

$$\begin{cases} u^0(x) = \dfrac{1}{6}(2u_{j-2} - 7u_{j-1} + 11u_j) \\ u^1(x) = \dfrac{1}{6}(-u_{j-1} + 5u_j + 2u_{j+1}) \\ u^2(x) = \dfrac{1}{6}(2u_j + 5u_{j+1} - u_{j+2}) \end{cases} \tag{5}$$

Then the numerical flux in five-point-stencil WENO scheme could be denoted with weights as

$$u_{j+1/2} = \omega_0u^0 + \omega_1u^1 + \omega_2u^2 \tag{6}$$

where ω is weight coefficient.

As for CAA problems, it is very important to simulate the short waves with a limited stencil scheme as much as possible. We may equate Eq. (6) and Eq. (1) to yield

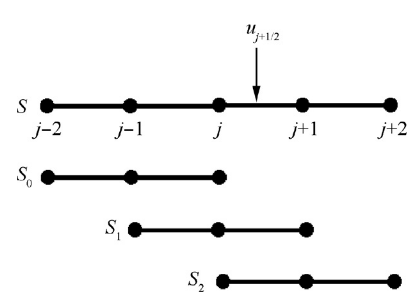

Fig. 1 Five-point-stencil WENO-JS scheme.