Chinese Journal of Aeronautics, (2018), **xxx**(xx): xxx–xxx

Chinese Society of Aeronautics and Astronautics
& Beihang University

Chinese Journal of Aeronautics

cja@buaa.edu.cn
www.sciencedirect.com

Nonlinear dynamics of a flapping rotary wing: Modeling and optimal wing kinematic analysis

Qiuqiu WEN [a,*], Shijun GUO [b], Hao LI [b], Wei DONG [a]

[a] *School of Aerospace, Beijing Institute of Technology, Beijing 100081, China*
[b] *Centre for Aeronautics, SATM, Cranfield University, Cranfield MK43 0AL, UK*

Received 17 April 2017; revised 15 May 2017; accepted 21 August 2017

KEYWORDS

Dynamic model;
Dynamic time-lag;
Flapping rotary wing;
Kinematics of wings;
Passive rotation;
Strike angle

Abstract The analysis of the passive rotation feature of a micro Flapping Rotary Wing (FRW) applicable for Micro Air Vehicle (MAV) design is presented in this paper. The dynamics of the wing and its influence on aerodynamic performance of FRW is studied at low Reynolds number ($\sim 10^3$). The FRW is modeled as a simplified system of three rigid bodies: a rotary base with two flapping wings. The multibody dynamic theory is employed to derive the motion equations for FRW. A quasi-steady aerodynamic model is utilized for the calculation of the aerodynamic forces and moments. The dynamic motion process and the effects of the kinematics of wings on the dynamic rotational equilibrium of FWR and the aerodynamic performances are studied. The results show that the passive rotation motion of the wings is a continuous dynamic process which converges into an equilibrium rotary velocity due to the interaction between aerodynamic thrust, drag force and wing inertia. This causes a unique dynamic time-lag phenomena of lift generation for FRW, unlike the normal flapping wing flight vehicle driven by its own motor to actively rotate its wings. The analysis also shows that in order to acquire a high positive lift generation with high power efficiency and small dynamic time-lag, a relative high mid-up stroke angle within 7–15° and low mid-down stroke angle within −40° to −35° are necessary. The results provide a quantified guidance for design option of FRW together with the optimal kinematics of motion according to flight performance requirement.

1. Introduction

The Micro Air Vehicle (MAV) has become an active research area due to the potentiality for the civil and military application.[1] The typical characteristics of MAV are small dimension (wing spans within 15 cm), low weight (gross take-off weight ranging from 100 to 200 g) and low flight speed (between 10 and 15 m/s). In recent two decades, a variety of MAV layouts, which mainly include fixed wing, rotary wing, and flapping

---

[*] Corresponding author.
E-mail address: wenqiuqiu82@bit.edu.cn (Q. WEN).
Peer review under responsibility of Editorial Committee of CJA.

Production and hosting by Elsevier

https://doi.org/10.1016/j.cja.2018.03.004

Please cite this article in press as: WEN Q et al. Nonlinear dynamics of a flapping rotary wing: Modeling and optimal wing kinematic analysis, *Chin J Aeronaut* (2018), https://doi.org/10.1016/j.cja.2018.03.004

Nomenclature

C_H, C_V	translational force coefficient along x_w axis and y_w axis
C_L, C_D	lift and drag coefficients
$\bar{C}_{L\_\mathrm{stab}}$	period average lift coefficient
C_r	rotational force coefficient
$\boldsymbol{C}_\mathrm{t}$	translational force coefficients
c	chord length of the wing
$\bar{c}$	mean chord length of the wing
$\mathrm{d}\boldsymbol{M}_\mathrm{q}$	aerodynamic torque of the above two forces
$\mathrm{d}\boldsymbol{F}_\mathrm{a}$	virtual mass force
$\mathrm{d}\boldsymbol{M}_\mathrm{a}$	virtual mass moment
$\boldsymbol{e}_{x,\mathrm{w}}$, $\boldsymbol{e}_{y,\mathrm{w}}$, $\boldsymbol{e}_{z,\mathrm{w}}$	unit vectors of right wing frame
$\boldsymbol{F}_\mathrm{aero}$, $\boldsymbol{M}_\mathrm{aero}$	total aerodynamic forces and moments
$\boldsymbol{F}_i^*$	inertia force of the ith rigid body
$\boldsymbol{F}_\mathrm{t}$, $\boldsymbol{F}_\mathrm{r}$	translational and rotational forces
f_F	flapping frequency
$\boldsymbol{I}_\mathrm{i}$	resulting mass moments of inertia matrices for each rigid body
i_stab	flapping period while FRW has been in the ERS
$\boldsymbol{M}_i^*$	inertia moment of the ith rigid body
$\boldsymbol{M}_\mathrm{mass}$	gravity moments due to the mass of wings
$\boldsymbol{M}_\mathrm{w}$	aerodynamic moments produced by flapping wings
m_i	mass of the ith rigid body
$O_\mathrm{b}x_\mathrm{b}y_\mathrm{b}z_\mathrm{b}$	body frame
$O_\mathrm{r}x_\mathrm{r}y_\mathrm{r}z_\mathrm{r}$	rotary plane frame
$O_\mathrm{w}x_\mathrm{w}y_\mathrm{w}z_\mathrm{w}$	wing-fixed frames
$\bar{P}_f$	average power output
$\bar{P}_{\mathrm{f\_stab}}$	power efficiency coefficient
Q_j^*	functions of generalized inertia force
R	span length of wing
$\boldsymbol{R}_\mathrm{br}$	rotation matrix from the body frame to rotation plane frame
$\boldsymbol{R}_\mathrm{bwR}$, $\boldsymbol{R}_\mathrm{bwL}$	rotation matrix from the body frame to right wing and left frames
$\boldsymbol{R}_\mathrm{Ib}$	transfer matrix from inertial frame to body frame
r_CP	location of the Centre of Pressure (CP) at a chord-wise location
S_w	the size of wing
T_F	flapping period
$\bar{t}$	time courses of wing motion during a flapping period
t_0	initial time at the beginning of one flapping period
$\boldsymbol{u}_i$	related quasi-velocities of coordinates
$\boldsymbol{v}_i$	velocities of the ith rigid body
$\boldsymbol{v}_\mathrm{w}(r)$	velocity of a chord-wise location on the wing
v_t	flapping velocity at the wingtip
$x_\mathrm{b}, y_\mathrm{b}, z_\mathrm{b}$	axes of the body frame
x_i	generalized coordinates of the five degrees of freedom
α_e	effective angle of attack of the wing
α_U, α_D	mid up-stroke and down-stroke angles
$\boldsymbol{\beta}_{ij}$	angular velocity coefficients
$\boldsymbol{\gamma}_{ij}$	velocity coefficients
$\Delta\gamma_\mathrm{w}$	flapping amplitude angle
$\Delta\alpha$	pitching amplitude
ϑ_w, γ_w	pitch angle and flap angle
λ_a, $\lambda_{a\omega}$	added mass force coefficients
$\bar{\mu}_{\mathrm{f\_stab}}$	nondimensional rotational velocity
ρ	density of the surrounding air
$\boldsymbol{\rho}_i$	reference vectors of the ith rigid body
ψ_{j0}	rotating speed
ψ_r	rotation angle of rotary base
$\boldsymbol{\omega}_i$	angular velocities of the ith rigid body

Subscripts

b	the body of FRW
i	the number of rigid bodies
j	the number of generalized coordinates
wL, wR	the left and right wing

wing, had been put forward. However, due to the extremely small dimension and high lift and efficiency requirements at low Reynolds numbers *Re*, few practical MAVs with load carrying capabilities has been accomplished. Research efforts for new and practical designs of MAVs have never been stopped.

In 2004, Vandenberghe et al.[2] employed the experimental method and found that a pair of wing flapping up and down can freely rotate spontaneously around the horizontal shaft as a critical frequency was exceeded. Based on this discovery, Guo et al.[3,4] proposed the design of Flapping Rotary Wing (FRW) flight vehicle as a new configuration of MAV. Similar concept was also proposed and applied in full-scaled helicopter rotor by Van Holten et al.[5] As shown in Fig. 1, a pair of anti-symmetrically mounted wings, which can flap along the vertical direction by a drive shaft, is fixed on the rotary rigid base. The thrust generated by the wings' vertically flapping motion drives them to rotate around the shaft, resulting in a flapping and simultaneously rotating kinematics. Combined with tuning the pitch angles of the wings asymmetrically in the up-stroke and down-stroke, the high lift force is produced to make FRW take-off and hover.

Recently, experimental works[6] were used to measure the force produced and proved that the lift from flapping rotary wing was larger than that from conventional rotary wing in the range of *Re* from 2600 to 5000. Wu et al.[7] conducted a computational fluid dynamics method to research the unsteady aerodynamic behavior of FRW. It is observed that the leading-edge vortex attached on the wing surface during the whole flapping period, which is the main reason for the high lift generation by FRW. Unlike the ordinary Flapping Wing (FW) flight vehicle which is driven by its own motor to rotate, the flapping rotary wing is driven by the aerodynamic force to rotate passively. Previous works on FRW have mostly assigned a constant rotation velocity by assuming an 'equilibrium' state. However, for a practical wing, the inertia forces associated with the complicated kinematics will essentially interact with the aerodynamic force production. The influence of the wing inertia and the dynamic process as the wing con-

Please cite this article in press as: WEN Q et al. Nonlinear dynamics of a flapping rotary wing: Modeling and optimal wing kinematic analysis, *Chin J Aeronaut* (2018), https://doi.org/10.1016/j.cja.2018.03.004