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Hysteresis behaviors of compressor rotating stall with cusp catastrophic model

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Abstract Rotating stall is a complex nonlinear dynamic phenomenon which is always characterized by catastrophe and hysteresis in high aerodynamic-loading compressor. Exploring the key contributing factors and characteristic rules of hysteresis is very important for compressor design and flow instability control. In this paper, a novel model method is proposed to analyze the hysteresis behaviors to extend the understanding of compressor rotating stall. The equilibrium states of compressor system under different conditions are first described based on Moore-Greitzer model. Then, through assessing the stability of the equilibrium points by Liapunov's theorem, the ratio of shutoff head to compressor characteristic semi-height is found to affect the stall hysteresis: the size of hysteresis loop will gradually decrease, even disappear with the increase of the ratio. Combing the effects of both the ratio and throttle coefficient, the hysteresis behaviors of compressor stall under multi-parameters can be found to be consistent with the topological properties of cusp catastrophic model by Thom's catastrophe theory. Finally, according to topological invariant rules, from the perspective of potential function, the equilibrium surface equation of compressor system is developed by standard cusp catastrophic model to describe the various hysteresis behaviors of compressor rotating stall along different control routes.

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1. Introduction

When the flow rate through a compressor is throttled gradually and the stall limit is reached, the essentially steady, axisymmetric flow becomes unstable. The result of this instability is very often manifested as a phenomenon known

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as rotating stall, which is an asymmetric phenomenon with one or several stall cells rotating at a fraction of the rotor speed while the overall mass flow rate remains nearly constant once the pattern is fully developed.[1,2]

Before stalling, two types of stall inception are often detected: model waves and spike.[3–7] The modal wave is characterized by the gradual growth of small-amplitude, essentially two dimensional, long wavelength disturbances. The spike is a three dimensional disturbance, which is localized at the tip region of a specific rotor in a multistage compressor, and has a length scale on the order of several blade pitches. According to the change of whole characteristic of the compressor in stall process, the types of rotating stall can also be divided into gradual stall and abrupt stall. For the early compressors,[8] in the stall process, the pressure rise often progressively changes as the flow rate increases and decreases. As the design loading of compressor blade increases, the stall-type of current mainstream compressors is generally abrupt stall; the pressure rise and the flow rate can be markedly reduced, and the efficiency can also drop sharply. In addition, a larger throttle opening is required to move the operating point from rotating stall to normal working condition than throttling at stall inception. These are the catastrophe and hysteresis of abrupt stall, the potentially serious threats to engine reliability.

For the catastrophe and hysteresis of compressor rotating stall, scholars have made some related researches. Day and Cumpsty[9] studied the hysteresis of compressor rotating stall through changing the design value of the flow coefficient, and found that the size of hysteresis loop will gradually decrease as the design value of the flow coefficient decreases. Copenhaver and Okiishi[10] tested the overall recoverability of a 10-stage compressor, and the results showed that higher shaft speeds cause low recoverability. The hysteresis of compressor rotating stall was first studied through numerical simulation by Choi et al.[11,12] They found that rotating stall at each operating point during recovery is stable, and rotating stall becomes unstable to be a transient after stall cells become too small to block the flow. Day et al.[13] introduced the blockage coefficient to assess the hysteresis of compressor rotating stall and found the compressor will recover from rotating stall when the blockage coefficient is less than 30%. From the point of structure stability, Abed et al.[14] and McCaughan[15] analyzed the characteristic of stall hysteresis with the help of bifurcation theory. They regarded the hysteresis of rotating stall as the system bifurcation. Liaw and Abed[16] applied the bifurcation theory to the active control of compressor stall inception, and eliminated the undesirable jump and hysteresis behavior of the uncontrolled system.

A series of studies have shown that the hysteresis behaviors of the compressor stall are affected by multi parameters, and determining the contributing factors and characteristic rule of hysteresis is essential to guide the design and control of the compressor. The catastrophe and hysteresis are related to the concept of "bifurcation" in mathematics. Currently, with the advancement of nonlinear dynamics, bifurcation theory has been applied to analyze the compressor rotating stall.[14–19] However, the bifurcation theory can only consider the effects of a single parameter. In 1983, the French mathematician Thom[20] first proposed catastrophe theory based on the bifurcation theory. The catastrophe theory can describe the effects of more control parameters on a system and has been widely used in the field of nonlinear dynamics.[21–25] Therefore, an idea to build a model to describe the hysteresis behaviors of compressor stall under the impact of multiple parameters is proposed based on the catastrophe theory.

This paper is organized as follows. Firstly, the equilibrium points of compressor system are determined based on Moore-Greitzer (M-G) model in Section 2. Then, in Section 3, the contributing factors of the stall hysteresis are analyzed and the physical mechanism of catastrophe and hysteresis is discussed through assessing the stability of the equilibrium points by Liapunov's theorem. Finally, according to topological invariant rules, the equilibrium surface equation of compressor is developed based on the standard cusp catastrophic model, and it is used to describe the diverse hysteresis behaviors of compressor rotating stall along different control routes in Section 4, which is then followed by conclusions (Section 5).

2. Equilibrium state of compressor system

The basic compressor system under study is shown in Fig. 1. This compressor system consists of a compressor with an inlet duct upstream and outlet duct downstream, followed by a plenum of relatively big volume and an exhaust pipe with a throttle valve at the exit. The simplest model which adequately describes the dynamics of rotating stall and surge in axial-flow compressor systems shown in Fig. 1 is the Moore-Greitzer model.[26,27] The full model is described in detail in the references and so we move straight to the simplified model.[28] The differential equations are as follows:

$$\frac{\mathrm{d}\phi}{\mathrm{d}\xi}=\left[-\frac{\psi-\psi_{\mathrm{c_0}}}{H}+1+\frac{3}{2}\phi\left(1-\frac{1}{2}J\right)-\frac{1}{2}\phi^3\right]\frac{H}{l_{\mathrm{c}}W} \tag{1}$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}\xi}=\frac{W}{4B^2l_{\mathrm{c}}}(\phi-\phi_{\mathrm{T}}) \tag{2}$$

$$\frac{\mathrm{d}J}{\mathrm{d}\xi}=J\left[1-\phi^2-\frac{1}{4}J\right]\frac{3aH}{(1+ma)W} \tag{3}$$

where the variable ϕ is nondimensional mass flow coefficient, which has been shifted so that zero mass flow actually occurs at $\phi=-1$, and rescaled with W. W is compressor characteristic semi-width. ψ is the nondimensional pressure rise of the compressor. Both of these variables are averaged over the annulus of the compressor. H is compressor characteristic semi-height. ξ refers to time for wheel to rotate one radian. The parameter $\psi_{\mathrm{c_0}}$ is the shutoff head, and it is proportional to the number of stages in the compressor. J is the square of the amplitude of the first mode of the rotating stall disturbance, so it only has physical meaning when it is positive. l_{c} refers to the total aerodynamic length of compressor and ducts. The parameter B in Eq. (2) is Greitzer's B parameter,[1,2] which determines the type of compressor instability. When the B is small, the flow usually develops into rotating stall. The variable ϕ_{T} represents the mass flow leaving the plenum and

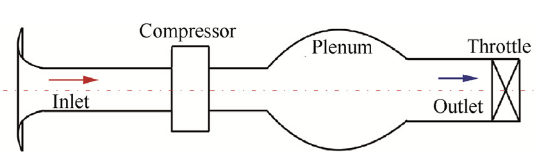

Fig. 1 Schematic of axial flow compressor system.