ARTICLE IN PRESS

Chinese Journal of Aeronautics, (2018), xxx(xx): xxx–xxx

Chinese Society of Aeronautics and Astronautics
& Beihang University

Chinese Journal of Aeronautics

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Extension of analytical indicial aerodynamics to generic trapezoidal wings in subsonic flow

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Received 13 January 2017; revised 26 May 2017; accepted 28 August 2017

KEYWORDS

Analytical approach;
CFD;
Compressible flow;
Gust response;
Indicial aerodynamics;
Trapezoidal wing

Abstract Analytical indicial aerodynamic functions are calculated for several trapezoidal wings in subsonic flow, with a Mach number $0.3 \leq Ma \leq 0.7$. The formulation herein proposed extends well-known aerodynamic theories, which are limited to thin aerofoils in incompressible flow, to generic trapezoidal wing planforms. Firstly, a thorough study is executed to assess the accuracy and limitation of analytical predictions, using unsteady results from two state-of-the-art computational fluid dynamics solvers as cross-validated benchmarks. Indicial functions are calculated for a step change in the angle of attack and for a sharp-edge gust, each for four wing configurations and three Mach numbers. Then, analytical and computational indicial responses are used to predict dynamic derivatives and the maximum lift coefficient following an encounter with a one-minus-cosine gust. It is found that the analytical results are in excellent agreement with the computational results for all test cases. In particular, the deviation of the analytical results from the computational results is within the scatter or uncertainty in the data arising from using two computational fluid dynamics solvers. This indicates the usefulness of the developed analytical theories.

1. Introduction

Indicial theory is a powerful mathematical tool that has been extensively employed in aerodynamics modelling (refer to Ref.[1] and references therein). Indicial theory asserts that the response of a linear time invariant system to an arbitrary input can be constructed by integrating a linear functional which involves the knowledge of the time dependent input signal and a kernel response. The kernel is an inherent characteristic

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Peer review under responsibility of Editorial Committee of CJA.

ELSEVIER Production and hosting by Elsevier

https://doi.org/10.1016/j.cja.2018.02.008
1000-9361

Please cite this article in press as: DA RONCH A et al. Extension of analytical indicial aerodynamics to generic trapezoidal wings in subsonic flow, *Chin J Aeronaut* (2018), https://doi.org/10.1016/j.cja.2018.02.008

of the system. Adding a nonlinear dependence of the functional on the input level[2] extends the capability of the model, allowing a certain class of model nonlinearity to enter the response. It is also important to observe that the traditional Volterra-Wiener theory[3,4] of nonlinear systems represents a subset of nonlinear indicial theory.

Researchers have followed three paths to address indicial aerodynamic modelling: an analytical path, a numerical path using high-fidelity CFD techniques, and an experimental path using measurements obtained in wind tunnel dynamic tests.

Analytical theories were derived under the assumption of a thin aerofoil in incompressible, irrotational, and two-dimensional flow. In the 1920s, Wagner[5] conducted a series of studies for the unsteady lift generated on an aerofoil due to abrupt changes in the angle of attack. The Wagner function describes the indicial built-up of the circulatory part of the lift, including the influence of the shed wake. Theodorsen[6] extended those studies by investigating the forces and moments on an oscillating aerofoil. The lift responses of an aerofoil penetrating sharp-edge and harmonically varying gusts were studied by Küssner[7] and Sears,[8] respectively. Further details on analytical theories of indicial aerodynamics and some recent developments, including the approach herein proposed, are given in Section 2.

Advances in computational power have allowed significant progress in the use of CFD techniques for modelling of nonlinear unsteady aerodynamics. To overcome the limitations of analytical indicial aerodynamics, restricted to linear flows and thin aerofoils, researchers investigated a few alternatives. The first attempts to directly determine the indicial response by CFD dated back to 1990s.[9] This approach has received widespread use (see Refs.[10,11] among many others) but still presents a number of difficulties, mostly associated with the numerical settings of an analysis and the reliability of unsteady results.

Other researchers have approached the modelling problem using indicial aerodynamics derived from wind tunnel dynamic tests and flight test measurements. For example, Refs.[12,13] applied linear indicial models to data from different testing facilities and different aircraft models. The identification of indicial models from flight test data was documented in Refs.[13,14] Nonlinear indicial responses were applied to a rolling 65° delta wing,[15] and in Ref.[16] to the prediction of a dynamically stalling wing.

A substantial portion of the work described in this paragraph was motivated by the increased manoeuvre capabilities and expanded flight envelopes of modern aircraft. More recently, under the NASA Aviation Safety Program, further research in unsteady modelling has been carried out at NASA Langley Research Center, and an excellent review of these methodologies is presented in Ref.[1].

The main contribution of this work is the derivation of an analytical indicial aerodynamics method that extends well-known theories, which are based on the assumption of thin aerofoils, to generic trapezoidal wings of finite span in subsonic flow. In particular, the paper is built around three objectives. The first is the formulation, application, and demonstration of a consistent analytical framework for predicting unsteady aerodynamic responses to arbitrary changes in the angle of attack and in the vertical component of the free-stream speed (gusts). The second objective places emphasis on the use of current state-of-the-art CFD modelling techniques, as provided by a widely-available open-source solver as well as an industrial-grade solver, for predicting unsteady viscous flows. The third objective draws a final assessment of the analytical model predictions considering the CFD-based unsteady aerodynamics uncertainty. A set of trapezoidal wings, with different Aspect Ratios (AR) and sweep angles, is tested at different flow conditions. In total, 24 unsteady aerodynamic test cases are executed for each methodology.

The paper continues in Section 2 with the analytical derivation of indicial aerodynamic functions valid for generic trapezoidal wings in subsonic flow. Section 3 summarizes the computational solvers and the appropriate techniques for the calculation of indicial aerodynamics. Then, results for four wing configurations and a set of flow conditions are presented and discussed in Section 4, highlighting the computational advantages and the related limitations where appropriate. Finally, conclusions are drawn in Section 5.

2. Analytical derivation of indicial functions

Built on previous work,[17] aerodynamic indicial functions for compressible subsonic flows are herein approximated by modification of the indicial functions for an incompressible flow. Prandtl-Glauert scalability[18] is used for the circulatory contribution, $\breve{C}_L(\tau)$, and piston theory[19] for the non-circulatory contribution, $\hat{C}_L(\tau)$. The lift coefficient is then found using the principle of superposition $C_L = \breve{C}_L(\tau) + \hat{C}_L(\tau)$.

Analytical formulae are derived combining the work of Queijo et al.[20] with that of Leishman.[21] The former describes the wing circulatory lift in incompressible flow, including the wake two-dimensional downwash and the tip vortices three-dimensional downwash.[22] The latter provides a theory for the calculation of the thin aerofoil lift in compressible flow, including Prandtl-Glauert theory for the circulatory terms and piston theory for the non-circulatory terms.

The circulatory lift build-up due to a unit sharp-edge gust with perturbation front parallel to the wing leading edge is then calculated by multiplying the lift response to a step change in the angle of attack with the ratio between Küssner and Wagner functions.[23] It is worth observing that the latter represents a fictitious angle of attack[24] and approximates the two-dimensional penetration effect within the "frozen gust" framework.[25]

The non-circulatory contribution drives the impulsive-like start of the flow response for any wing shape and is followed by a short yet complex region where outgoing and incoming acoustic waves intersect.[19] The circulatory contribution drives the subsequent lift build-up until steady state convergence. As the asymptotic lift value provided by Queijo et al.[20] is originally deemed inaccurate, it is here obtained via simplified lifting-line theory.[26] An alternative for fine-tuning the asymptotic value is to use available numerical or experimental data[27], so that viscous effects may statically be recovered in the absence of significant flow separation.[28] With identical reference flow conditions, the initial lift coincides for both tuned and untuned cases but later develops with a different rate.

For swept wings, the entry delay relative to each section is geometrically known and considered when obtaining the lift build-up due to a unit sharp-edge gust with perturbation front normal to the reference airflow.