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Stagnation temperature effect on the conical shock with application for air

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Abstract The aim of this work is to realize a new numerical program based on the development of a mathematical model allowing determining the parameters of the supersonic flow through a conical shock under hypothesis at high temperature, in the context of correcting the perfect gas model. In this case, the specific heat at constant pressure does not remain constant and varies with the increase of temperature. The stagnation temperature becomes an important parameter in the calculation. The mathematical model is presented by the numerical resolution of a system of first-order nonlinear differential equations with three coupled unknowns for initial conditions. The numerical resolution is made by adapting the higher order Runge Kutta method. The parameters through the conical shock can be determined by considering a new model of an oblique shock at high temperature. All isentropic parameters of after the shock flow depend on the deviation of the flow from the transverse direction. The comparison of the results is done with the perfect gas model for low stagnation temperatures, upstream Mach number and cone deviation angle. A calculation of the error is made between our high temperature model and the perfect gas model. The application is made for air.

1. Introduction

The problem of supersonic flow around a cone plays a practical importance in aerospace applications, as the majority of gears are formed by a conical shape approaching to a circular shape.[1–5] It can be found at the level of projectiles, in particular at the nose of an airplane or missile, or even at the air intake of a supersonic engine, where it requires an adaptation of the air intake in order not to have a shock wave reflection in the motor. The problem of supersonic flow around a cone presents a case of a real Three-Dimensional (3D) flow obtained by rotating a Two-Dimensional (2D) dihedral around an axis of revolution. It can be solved by the use of conservation equations in a differential form.[2,4,6,7] The cylindrical coordinates in this case are adapted for the presentation of a mathematical model.

Some authors have used curved forms of cones to have a progressive flow or a straight line.[1–7] The pointed shape of a cone at the leading edge is very interesting in order to study

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Nomenclature

L	longitudinal length of the cone
V_r	radial component of the velocity vector
V_θ	transverse component of the velocity vector
r	polar ray
θ	deviation of the radial velocity
ρ	density
V	velocity
p	pressure
T	temperature
a	sound velocity
H	enthalpy
T_0	stagnation temperature
R	thermodynamic constant of air
γ	specific heats ratio
Ma	Mach number
μ	Mach angle
ρ_1/ρ_{01}	isentropic density ratio before the shock
p_1/p_{01}	isentropic pressure ratio before the shock
T_1/T_0	isentropic temperature ratio before the shock
ψ	deviation of the flow just after the shock
φ	angle of the dihedron or the cone
θ_C	deviation of cone surface
θ_S	deviation of conical shock
ρ_2/ρ_{02}	isentropic density ratio after the shock
p_2/p_{02}	isentropic pressure ratio after the shock
T_2/T_0	isentropic temperature ratio after the shock
Ma_{2n}	downstream Mach number for a normal shock
χ	accurary of the numerical calculation for the downstream Mach number
x	variable change
f, g, h	condensed functions used for the Runge Kutta method
k_i, l_i, m_i	terms used in the numerical Runge Kutta method
Δx	numerical step of the Runge Kutta method
α	velocity vector deviation
Drag	drag force on the cone surface
D	normalized drag force on the cone surface
ρ_C/ρ_{02}	isentropic density ratio on the cone surface
p_C/p_{02}	isentropic pressure ratio on the cone surface
T_C/T_0	isentropic temperature ratio on the cone surface
p_{02}/p_{01}	totale pressures ratio through the shock
ΔS_{21}	entropy jump through the shock
C	dimensional distance of the adapted air intake
d	adapted air intake distance
Radius	radius of the intake
ε	relative error caused by the PG model compared to the HT model
τ	accurary of the Runge Kutta method

Subscripts

1	upstream condition of the shock
2	downstream condition of the shock
0	stagnation parameter
C	parameter on the cone surface
n	normal parameter to shock
G	left parameter of an interval
D	right parameter of an interval
Surface	surface of the cone or the dihedron

the problem of the possibility to have attached shock waves. We find the first study on the determination of conical shocks for a calorically and thermally perfect gas in the Refs.[8–10] This study assumes that the specific heat c_p is a constant value and does not vary with the temperature. It gives good results only if the three parameters Ma_1, T_0, and θ_C are small, which do not exceed respectively about 2.00, 240 K, and 20°. If Ma_1 begins to gradually exceed 2.00 to the supersonic limit taken at $Ma_1 = 5.00$, or when T_0 begins to exceed 240 K up to the limit of 3550 K (since the application is for air), or when θ_C gradually exceeds 20°, which is the case for the majority of aerospace applications, the results presented in these references are quite far from reality, since the physical behavior of gas changes, and it becomes calorically imperfect and thermally perfect[11,12] or gas at high temperature, which requires thinking about corrections to these results by looking for a mathematical model that meets our needs. In this case, the specific heat c_p becomes a function of temperature, and the energy conservation equation changes completely. This model is called as model at high temperature, lower than the threshold of the molecules dissociation. The new model presented essentially depends on the stagnation temperature which becomes an important parameter, in addition to the parameters of the perfect gas model.

For the function c_p and for air, one finds a series of tabulated values at temperatures between 55 K and 3550 K (limit the temperature to avoid the air dissociation).[1,13,14] Then in this case, a polynomial interpolation was made to these values in order to find an appropriate analytical form. A 9th-degree polynomial is chosen, giving a maximum error of less than 0.01% between the value given by the polynomial and the table for any temperature in the shown range.[11,12]

The aim of this work is to determine a mathematical model corresponding to the need for high temperature to make corrections to the results given by the Perfect Gas (PG) model[1,3–5,8–10] and to the supersonic flow around a right cone with zero incidence when the three parameters (Ma_1, θ_C, T_0) are high but lower than the dissociation threshold of the molecules. Therefore, the problem, in the second stage, is to develop a new numerical calculation program allowing to determine new parameters through a conical shock like the shock deviation and parameters through and after the shock (Ma_2, p_2/p_1, T_2/T_1, ρ_2/ρ_1, ψ, p_{02}/p_{01}, and ΔS_{21}), and thermodynamic parameters on the cone surface (Ma_C, T_C/T_0, ρ_C/ρ_{02}, and p_C/p_{02}) and in any direction after the shock. These results must be used to determine, for example, other aerodynamic characteristics, such as the air intake adaptation distance and the shock wave drag delivered by the supersonic flow on the cone. All these results are a function of (Ma_1, T_0, and θ_C) for our High Temperature (HT) model, which can be considered as a generalization of the PG model.

The first step consists of determining the shock deviation θ_S corresponding to Ma_1, θ_C, and T_0. Since the application is for air, all remaining and necessary parameters can then be determined through appropriate procedures. Let solve nonlinear