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Prediction of fatigue crack growth rate for small-sized CIET specimens based on low cycle fatigue properties

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Abstract Based on experiments of low cycle fatigue for 5083-H112 aluminum alloy, two energy-based predictive models have been introduced to predict the fatigue crack growth behaviors of traditional Compact Tension (CT) and small-sized C-shaped Inside Edge-notched Tension (CIET) specimens with different thicknesses and load ratios. Different values of the effective stress ratio U are employed in the theoretical fatigue crack growth models to correct the effect of crack closure. Results indicate that the two predictive models show different capacities of predicting the fatigue crack growth behaviors of CIET and CT specimens with different thicknesses and load ratios. The accuracy of predicted results of the two models is strongly affected by the method for determination of the effective stress ratio U. Finally, the energy-based Shi&Cai model with crack closure correction by means of Newman's method is highly recommended in prediction of fatigue crack growth of CIET specimens via low cycle fatigue properties.

1. Introduction

The fatigue crack propagation rate per cycle, da/dN, governed by the stress intensity factor range, ΔK, is commonly applied to represent the fracture behavior of a cracked body subjected to cyclic loading. Taking into account local cyclic plastic deformation around the crack tip, the fatigue crack growth behavior can be predicted by the low cycle fatigue property of a material in conjunction with a description of the stress and strain field ahead of the crack tip and an appropriate failure criterion.

Different failure criteria such as critical stress, plastic strain ahead of the crack tip,[1] the magnitude of crack tip damage accumulation ahead of the crack tip,[2–7] and strain energy[5–13] have been used in past fatigue crack growth models. These energy-based criteria are mainly based on the critical level of energy dissipation within the material ahead of the crack tip, and it is found that they are more accurate than other failure criteria in predicting fatigue crack growth behaviors. An

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important fact generally observed in fatigue crack growth experiments is that the specimen geometry and load ratio can remarkably affect the fatigue crack growth law because of the crack closure effect.[14–17] Only Shi et al.[7] introduced an effective stress ratio U to quantify the crack closure effect in a fatigue crack growth model. Here, the used effective stress ratio U can only eliminate the effect of the load ratio on the fatigue crack growth rate. For different specimen geometries, the transverse T-stress is another factor which may alter the cyclic plastic zone size and further affect the crack closure behavior.[16,18] Bao et al.[19] conducted a group of experiments on the fatigue crack growth rate of 5083-H112 aluminum alloy by using traditional Compact Tension (CT) and small-sized C-shaped Inside Edge-notched Tension (CIET) specimens, and the resulted fatigue crack growth curves showed an outstanding difference between two specimen geometries.

The present work aims to predict the fatigue crack growth data reported in Ref. 19, according to two types of energy-based fatigue crack growth models based on low cycle fatigue properties by introducing the effective stress ratio U determined by different methods.

2. Fatigue crack growth models

2.1. Cyclic stress and strain fields ahead of crack tip

For a crack body subjected to a remote external load, the classical HRR[20,21] solution is commonly employed to describe the stress and strain fields in the vicinity of the crack tip under a plane stress condition. By introducing the plastic superposition principle,[22] the cyclic stress and strain fields ahead of the crack tip under small-scale yielding can be obtained from the HRR solution[23] as follows:

$$\begin{cases} \Delta\sigma = 2\sigma_{\mathrm{yc}}\left(\frac{\Delta K^2}{4\alpha_{\mathrm{c}}\sigma_{\mathrm{yc}}^2 I_{n_{\mathrm{c}}} r_c}\right)^{\frac{1}{n_{\mathrm{c}}+1}}\tilde{\sigma}_\theta \\ \Delta\varepsilon = \Delta\varepsilon_{\mathrm{e}} + \Delta\varepsilon_{\mathrm{p}} \end{cases} \tag{1}$$

in which

$$\begin{cases} \Delta\varepsilon_{\mathrm{e}} = \frac{2\sigma_{\mathrm{yc}}}{E}\left(\frac{\Delta K^2}{4\alpha_{\mathrm{c}}\sigma_{\mathrm{yc}}^2 I_{n_{\mathrm{c}}} r_{\mathrm{c}}}\right)^{\frac{1}{n_{\mathrm{c}}+1}}(\tilde{\sigma}_\theta - v\tilde{\sigma}_r) \\ \Delta\varepsilon_{\mathrm{p}} = \frac{2\alpha_{\mathrm{c}}\sigma_{\mathrm{yc}}}{E}\left(\frac{\Delta K^2}{4\alpha_{\mathrm{c}}\sigma_{\mathrm{yc}}^2 I_{n_{\mathrm{c}}} r_{\mathrm{c}}}\right)^{\frac{n_{\mathrm{c}}}{n_{\mathrm{c}}+1}}(\tilde{\sigma}_\theta - 0.5\tilde{\sigma}_r) \end{cases} \tag{2}$$

where E is elastic modulus, $\Delta\sigma$ and $\Delta\varepsilon$ are the stress and strain ranges, respectively. ΔK is the stress intensity factor range, and (r, θ) are the polar coordinates centered at the crack tip. σ_{yc} is the reference cyclic yield stress, α_{c} is the cyclic strain hardening coefficient in the Ramberg-Osgood relationship,[24] and n_{c} is the cyclic strain hardening exponent but is the reciprocal of the exponent in the Ramberg-Osgood model. $\tilde{\sigma}_\theta$, $\tilde{\sigma}_r$, and $I_{n_{\mathrm{c}}}$ are dimensionless distribution functions only related to n_{c} and tabulated by Shih.[25] r_{c} is the cyclic plastic zone under mode I loading considering the stress redistribution and the strain hardening effect, and its expression under the plane stress condition can be described as[13]

$$r_{\mathrm{c}} = \frac{\Delta K^2}{8(1+n_{\mathrm{c}})\pi\sigma_{\mathrm{yc}}^2}\left(1 + \frac{3}{2}\sin^2\theta + \cos\theta\right) \tag{3}$$

2.2. Energy-based fatigue crack growth models

In the research of low cycle fatigue behavior, the well-known Manson-Coffin model is commonly applied to describe the relationship between the strain amplitude, $\Delta\varepsilon/2$, and the number of reversals to failure, $2N_{\mathrm{f}}$, in the following form:

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_{\mathrm{e}}}{2} + \frac{\Delta\varepsilon_{\mathrm{p}}}{2} = \frac{\sigma'_{\mathrm{f}}}{E}(2N_{\mathrm{f}})^b + \varepsilon'_{\mathrm{f}}(2N_{\mathrm{f}})^c \tag{4}$$

where σ'_{f} and $\varepsilon'_{\mathrm{f}}$ are the fatigue coefficients of strength and ductility, while b and c are the fatigue strength and ductility exponents, which can be easily determined from low cycle fatigue test data.

By taking into consideration the total ductility loss of a material within a cyclic plastic zone, Pandey and Chand[12,13] developed a fatigue crack growth model based on the low cycle fatigue property as follows:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \frac{(1-n_{\mathrm{c}})\tilde{\sigma}_\theta(\tilde{\sigma}_\theta - 0.5\tilde{\sigma}_r)}{2EI_{n_{\mathrm{c}}}\sigma'_{\mathrm{f}}\varepsilon'_{\mathrm{f}}}(\Delta K - \Delta K_{\mathrm{th}})^2 \tag{5}$$

where ΔK_{th} is the threshold stress intensity factor range. Here, the crack closure effect is not considered in the Pandey&Chand model. Similarly, Shi and Cai[5] proposed another energy-based fatigue crack growth model according to the equivalence of plastic strain energy within the cyclic plastic zone, and it will be hereafter referred to as the Shi&Cai model. This model is described as follows:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \frac{r_{\mathrm{c}} - \rho_{\mathrm{c}}}{N_{\mathrm{f}}} \tag{6}$$

in which

$$\begin{cases} r_{\mathrm{c}} = \frac{\Delta K^2}{4(1+n_{\mathrm{c}})\pi\sigma_{\mathrm{yc}}^2} \\ \rho_{\mathrm{c}} = \frac{\Delta K_{\mathrm{th}}^2}{4(1+n_{\mathrm{c}})\pi\sigma_{\mathrm{yc}}^2} \end{cases} \tag{7}$$

$$N_{\mathrm{f}} = \frac{1}{2}\left[\frac{\alpha_{\mathrm{c}}\sigma_{\mathrm{yc}}^2(\tilde{\sigma}_\theta - 0.5\tilde{\sigma}_r)}{E\sigma'_{\mathrm{f}}\varepsilon'_{\mathrm{f}}} \cdot \frac{(1+n_{\mathrm{c}})\pi}{\alpha_{\mathrm{c}} I_{n_{\mathrm{c}}}} \cdot \frac{r_{\mathrm{c}}}{r_{\mathrm{c}} - \rho_{\mathrm{c}}} \ln\left(\frac{r_{\mathrm{c}}}{\rho_{\mathrm{c}}}\right)\right]^{\frac{1}{b+c}} \tag{8}$$

where ρ_{c} is the cyclic plastic zone corresponding to ΔK_{th}. To eliminate the effect of crack closure on the fatigue crack growth, Shi et al.[7] introduced an effective stress ratio U proposed by Antunes et al.[26] and Codrington et al.[27] into the Shi&Cai model, which is

$$U = 0.446 + 0.373R + 0.2R^2 \quad R \geqslant 0 \quad \text{Plane stress} \tag{9}$$

Here, the effective stress ratio U is deduced from a rigid perfectly plastic strip-yield model, and is only related by the load ratio R. According to the correction of crack closure using the effective stress ratio U, the Shi&Cai model can be amended by replacing the cyclic plastic zones r_{c} and ρ_{c} with the effective cyclic plastic zones r_{eff} and ρ_{eff} as shown in the following equation:

$$\begin{cases} r_{\mathrm{eff}} = \frac{(U\Delta K)^2}{4(1+n_{\mathrm{c}})\pi\sigma_{\mathrm{yc}}^2} \\ \rho_{\mathrm{eff}} = \frac{(U\Delta K_{\mathrm{th}})^2}{4(1+n_{\mathrm{c}})\pi\sigma_{\mathrm{yc}}^2} \end{cases} \tag{10}$$

In fact, not only the load ratio R but the specimen geometry characterized by the transverse T-stress may also affect the crack closure and further affect the fatigue crack growth behavior. In the work of Bao et al.,[19] the classical plastic

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