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Singularity analysis for single gimbal control moment gyroscope system using space expansion method

Yunhua WU [a,*], **Feng HAN** [a,b], **Bing HUA** [a], **Zhiming CHEN** [a], **Feng YU** [a]

[a] *School of Astronautics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China*
[b] *School of Astronautics, Harbin Institute of Technology, Harbin 150001, China*

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Abstract Control Moment Gyroscope (CMG) is an effective candidate for agile satellites and large spacecraft attitude control because of its powerful torque amplification capability. The most serious situation, however, in using CMG is the inherent geometric singularity problem, where there's no torque output along a particular direction. Space expansion method has been proposed in this work for the singularity analysis. Based on inverse mapping transformation, an expanded Jacobian matrix which is a full rank square matrix is obtained. The singular angle sets of the 3-parallel cluster and pyramid cluster are distinguished using space expansion method. An effective hybrid steering strategy, able to deal with the elliptic singularity, is further proposed. Simulation results demonstrate the excellent performance of the proposed steering logic compared to the generalized singular robust logic and pseudo inverse logic in terms of energy consumption and torque error.

1. Introduction

Single Gimbal Control Moment Gyroscope (SGCMG) is a kind of momentum exchange actuator for spacecraft attitude control, due to its powerful momentum storage capacity and amazing torque amplification ability over Reaction Wheel (RW), magnetic torque, etc. The SGCMG, owing to its efficiency, plays a significant role in attitude control of large spacecraft and agile spacecraft. Larger angle maneuver, multi target acquisition and precise pointing are the major characteristics for the next generation earth observation and imaging satellite.[1] The World View satellites launched by the Digital Globe and Ball Aerospace and Technologies Corporation could capture High Resolution (HR) images, of which the World View 4 is the most advanced throughout the history.[2] Launched in 2001 the World View 1 is the first commercial HR imaging satellite equipped with the CMGs, which can only capture the images with 0.5 m panchromatic resolution and 2 m multispectral resolution. While the World View 4 launched on Nov. 11, 2016 advances a great step and is capable for

* Corresponding author.
E-mail address: yunhuawu@nuaa.edu.cn (Y. WU).
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0.31 m panchromatic resolution and 2 m multispectral resolution images.

The most serious situation, however, in using CMG is the inherent geometric singularity, in which there is no torque output along a specific direction.[3,4] Many theories and strategies have been employed to distinguish the singularity problem and to design CMG steering logics. Null motion,[5] Singular Value Decomposition (SVD),[6,7] differential geometry and topology,[6–8] and others are applied for singularity analysis. The Pseudo-Inverse Logic (PIL) and Null motion, although can steer CMG with no torque errors, fail to deal with singularity. While this tough problem can be tackled by the singular robust steering logics with some torque errors added, which actually is a compromise between the preciseness and robustness. The SVD method gives researchers insight into the torque space and gimbal rate space and explain the mechanism of some logics.

A Space Expansion Method (SEM) is proposed in this work to deal with the singularity problem. The SEM would expand the CMG gimbal movement to n dimensional space and achieve a square Jacobian Matrix which will simplify the procedure to find the corresponding gimbal rate for the given torque command. Besides, two different kinds of transformations, Inverse Mapping Transformation (IMT) and Constant Magnitude Transformation (CMT), are introduced to construct an Expanded Jacobian Matrix. Based on SEM, the singularity of the 3-parallel cluster and pyramid cluster is analyzed with a resulted singular gimbal angle sets. What's more, a hybrid steering strategy is proposed for the parallel CMG cluster and pyramid CMG cluster, which behaves more energy saving and more precise by comparing with the generalized singular robust logic.

The rest of this paper is briefly outlined as follows: Section 2 summarizes the CMG dynamic model and steering logics. The space expansion method is represented in Section 3 with some illustrative examples. Section 4 argues the singularity problem and compares the space expansion method with other methods. A simple steering logic is represented in Section 5 to illustrate the advantage and potential of SEM with some simulations. Finally, Section 6 concludes the whole work with some suggestion for future research.

2. CMG dynamics and steering logics

Specific space missions require different CMG configurations. A cluster with more than 3 CMG units is regarded as redundant configuration, and is qualified for 3 axes attitude control, while the non-redundant system also plays a significant role, which may be expanded to get redundant arrays. The pyramid array is the most studied system[9–14] and has a mini-redundancy. The other non-redundant cluster, 3-parallel configuration, is also investigated in this paper due to its simple dynamic mechanism. For the above clusters there exist different steering logics. And their mechanism will be examined and demonstrated by SVD method.

2.1. Parallel and pyramid CMG clusters

Although the non-redundant cluster is not capable for 3-axis attitude control, most agile imaging satellite such as the earth observing satellite only have a larger torque requirement in roll and pitch axes, which inspires scientists to employ 2 or 3 CMGs with the gimbal axis parallels with roll/pitch axis and a RW for yaw axis control.

According to physical mechanism of CMG, the gimbal axis $\boldsymbol{g}$ and the flywheel momentum $\boldsymbol{h}$ are orthogonal to each other. Therefore, a gimbal frame $\{\boldsymbol{g}_i, \boldsymbol{h}_i, \boldsymbol{f}_i\}$ can be derived for the ith CMG in a cluster, where $\boldsymbol{f}$ is the unit vector and represents the torque direction. To simplify the dynamic model, all the quantities should be represented in the Spacecraft Reference Frame (SRF).

The 3-parallel cluster structure is shown in Figs. 1 and 2 depicts the flywheel momentum vectors in X-Y plane. The momentum of the whole system can be written as

$$\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2, \boldsymbol{h}_3] = \begin{bmatrix} \cos\delta_1 & \cos\delta_2 & \cos\delta_3 \\ \sin\delta_1 & \sin\delta_2 & \sin\delta_3 \end{bmatrix} \tag{1}$$

where $\boldsymbol{h}_i$ is ith CMG flywheel momentum represented in the SRF and δ_i is the corresponding gimbal angle. We suppose that each CMG flywheel momentum is 1 in terms of magnitude. Applying differential to Eq. (1) we could get the torque equation

$$\begin{aligned} \dot{\boldsymbol{H}} &= \frac{\partial \boldsymbol{H}}{\partial \boldsymbol{\delta}}\dot{\boldsymbol{\delta}} = [\boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{f}_3]\dot{\boldsymbol{\delta}} = \boldsymbol{J}\dot{\boldsymbol{\delta}} \\ &= \begin{bmatrix} -\sin\delta_1 & -\sin\delta_2 & -\sin\delta_3 \\ \cos\delta_1 & \cos\delta_2 & \cos\delta_3 \end{bmatrix} \begin{bmatrix} \dot{\delta}_1 \\ \dot{\delta}_2 \\ \dot{\delta}_3 \end{bmatrix} \end{aligned} \tag{2}$$

where $\dot{\boldsymbol{\delta}} = [\dot{\delta}_1, \dot{\delta}_2, \dot{\delta}_3]^{\mathrm{T}}$ is the gimbal angle rate vector, $\boldsymbol{f}_i = \mathrm{d}\boldsymbol{h}_i/\mathrm{d}\delta_i$ is the CMG unit torque direction vector and $\boldsymbol{J}^{2\times3} = \partial\boldsymbol{H}/\partial\boldsymbol{\delta}$ is the Jacobian Matrix. It can be found that $\boldsymbol{J}$ is not a square matrix, and it's difficult to figure out the gimbal rate for the given torque command.

The most classical 4-CMG system is the pyramid configuration, as illustrated in Fig. 3. The gimbal axes are normal to the four side faces of a pyramid respectively. Similarly, we could get the momentum as well as the torque equation of the pyramid cluster, and they are given in the following Eqs. (3) and (4).

$$\boldsymbol{H} = \begin{bmatrix} -c\beta s_1 & -c_2 & c\beta s_3 & c_4 \\ c_1 & -c\beta s_2 & -c_3 & c\beta s_4 \\ s\beta s_1 & s\beta s_2 & s\beta s_3 & s\beta s_4 \end{bmatrix} \tag{3}$$

$$\dot{\boldsymbol{H}} = \begin{bmatrix} -c\beta c_1 & s_2 & c\beta c_3 & -s_4 \\ -s_1 & -c\beta c_2 & s_3 & c\beta c_4 \\ s\beta c_1 & s\beta c_2 & s\beta c_3 & s\beta c_4 \end{bmatrix} \begin{bmatrix} \dot{\delta}_1 \\ \dot{\delta}_2 \\ \dot{\delta}_3 \\ \dot{\delta}_4 \end{bmatrix} \tag{4}$$

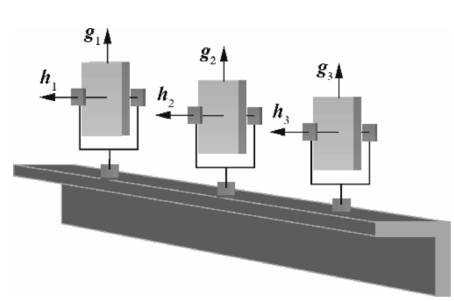

Fig. 1 3-parallel CMG cluster.

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