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Geometrically exact nonlinear analysis of pre-twisted composite rotor blades

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Abstract Modeling of pre-twisted composite rotor blades is very complicated not only because of the geometric non-linearity, but also because of the cross-sectional warping and the transverse shear deformation caused by the anisotropic material properties. In this paper, the geometrically exact nonlinear modeling of a generalized Timoshenko beam with arbitrary cross-sectional shape, generally anisotropic material behavior and large deflections has been presented based on Hodges' method. The concept of decomposition of rotation tensor was used to express the strain in the beam. The variational asymptotic method was used to determine the arbitrary warping of the beam cross section. The generalized Timoshenko strain energy was derived from the equilibrium equations and the second-order asymptotically correct strain energy. The geometrically exact nonlinear equations of motion were established by Hamilton's principle. The established modeling was used for the static and dynamic analysis of pre-twisted composite rotor blades, and the analytical results were validated based on experimental data. The influences of the transverse shear deformation on the pre-twisted composite rotor blade were investigated. The results indicate that the influences of the transverse shear deformation on the static deformation and the natural frequencies of the pre-twisted composite rotor blade are related to the length to chord ratio of the blade.

1. Introduction

Modeling a pre-twisted composite rotor blade is complicated, not only because of the geometric non-linearity, but also because of the cross-sectional warping and the transverse shear deformation caused by anisotropic material properties. A pre-twisted composite rotor blade was generally simplified into a one-dimensional beam analysis and two-dimensional cross-sectional analysis. In the one-dimensional beam analysis, either a moderate deflection beam theory or a large deflection beam theory was adopted. The moderate deflection beam the-

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ory neglects the higher-order terms by an ordering analysis so that the strain-displacement relations and the relationship between the deformed coordinate and the undeformed coordinate can be expressed by the displacements and their derivatives. Hence, the established equations of motion are very complex, containing many terms. The large deflection beam theory does not use any restrictive hypothesis regarding deformations or angles other than a small-strain assumption. Thus, it can be used to solve problems of large deflections. In the two-dimensional cross-sectional analysis, a direct analysis method or a finite element method can be used. The direct analysis method is based on shell theory and usually is used to simplify a realistic composite blade to the form of a thin-walled or thick-walled beam. Hence, the direct analysis method is often used for preliminary design and/or optimization of composite blade. On the other hand, a finite element method can model realistic composite blades (e.g., with foam-filled core, leading-edge weight, etc.) and is often used in detailed design.

Hong and Chopra[1] simplified a composite blade as a single-cell laminated beam to establish a composite blade model in which strain-displacement relations based on the moderate deflections by Hodges and Dowell were used, and the shear stress through the thickness of the beam was neglected. Based on the modeling of Hong and Chopra, Panda and Chopra[2] studied the effect of elastic couplings on the dynamic characteristics of a blade in forward flight. Smith and Chopra[3] improved the model of Hong and Chopra[1] by including the torsion-related out-of-plane warping, transverse shear deformation and two-dimensional ply elasticity. By using the improved model, Smith and Chopra[4,5] established a hingeless rotor model to study the effects of elastic couplings caused by the tailoring of composite materials for improving the aeroelastic stability of rotor and reducing the vibratory load of rotor. Based on the aforementioned research, Chopra et al.[6] established a thick-walled composite blade model for arbitrary cross-sectional shape and material properties.

Hodges[7] established the geometrically exact nonlinear equations of motion for a one-dimensional beam by using Hamilton's principle. Rodrigues parameters were used to express the finite rotation matrix of beam cross section. These established equations of motion can analyze large deflections. They included unknown variables for displacement, rotation, cross-sectional stress resultants, moment resultants, and linear and angular momenta. Hodges et al.[8] established a generalized classical beam model which is suitable for inhomogeneous, anisotropic, slender, prismatic beams by combining the rotation tensor decomposition method and the variational asymptotical method. The model consists of one-dimensional geometrically exact nonlinear beam analysis and two-dimensional linear cross-sectional analysis. Except for the small strain assumption, no deformation and angle assumptions were adopted in the model. The transverse shear deformation, arbitrary cross-sectional warping and elastic couplings were considered in the model. Hence, the model can analyze large deflections. Cesnik[9] added the initial twist and curvature into the model established by Hodges et al.[8] and established both generalized classical and generalized Timoshenko beam models for analysis of both slender and arbitrary beams, respectively, by using different coordinate systems. Popescu[10] pointed out that the generalized Timoshenko beam model established by Cesnik[9] was not asymptotically correct, and established an approach to transform the generalized classical beam model into the asymptotically correct generalized Timoshenko beam model for prismatic beams by using the equilibrium equations and a minimization method. Yu[11] further improved the generalized Timoshenko beam model established by Popescu[10] by adding the initial twist and curvature into the model, using the more feasible perturbation method instead of a minimization method in the transforming process and eliminating the limitation that the beam axis must be chosen at the centroid of cross section. However, in the process of transforming the generalized classical beam model into the generalized Timoshenko beam model, Yu[11] neglected some higher-order terms in the expression of strain energy which may have nonnegligible effects for some beam structures. Based on the aforementioned research, Hodges[12] systematically improved the model for composite beams with arbitrary cross-sectional shape and material properties. Ho et al.[13,14] established the perturbation method including all transformation terms of the strain energy on the basis of research by Yu[11], insuring the accuracy of the analysis.

Friedmann et al.[15–17] established a composite blade model for arbitrary cross-sectional shape and material properties based on finite element method. The model was based on the moderate deflection beam theory and considered the pre-twist, transverse shear deformation and out-of-plane warping. Yin and Xiang[18,19] studied the effects of transverse shear and warping on rotor aeroelastic stability by using the method of Friedmann et al.[15–17] and analyzed the influence of elastic couplings on the aeroelastic response and loads of composite rotor in forward flight. Friedmann et al.[20] introduced the two-dimensional cross-sectional analysis model developed by Hodges et al. into the composite blade model which was based on the moderate deflection beam theory. Based on the improved formulations of Friedmann et al.[20], Shi et al.[21] developed a modified 14 degree of freedoms beam element to predict the blade structural deformation.

In the above mentioned studies, several differences can be observed: (A) The cross section analyses by Chopra et al.[1,3,6] belong to the direct analysis method, and Hodges et al.[12–14] and Friedmann et al.[15–17] used the finite element method to perform the two-dimensional cross section analysis; (B) The one-dimensional beam analyses by Chopra et al.[1,3,6] and Friedmann et al.[15–17] were based on the moderate deflection beam theory, and Hodges et al.[12–14] used the large deflection beam theory to analyze the one-dimensional beam; (C) Chopra et al.[1,3,6] and Friedmann et al.[15–17] assumed the specific distributions of cross-sectional warping for composite blade, and Hodges et al.[12–14] transformed the geometrically nonlinear elastic analysis into the two-dimensional cross section analysis and the one-dimensional geometrically exact nonlinear analysis by using the variational asymptotical method and determined the arbitrary warping of beam cross section during the transforming process. Therefore, compared with the models of composite blade by Chopra et al.[1,3,6] and Friedmann et al.[15–17], the generalized Timoshenko beam model by Hodges et al.[12–14] is more suitable for modeling composite blades.

In this paper, the geometrically exact nonlinear modeling for the generalized Timoshenko beam with arbitrary cross-sectional shape, generally anisotropic material behavior and large deflection has been presented based on the Hodges' method. The concept of decomposition of rotation tensor