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# Remaining useful life prediction based on variation coefficient consistency test of a Wiener process

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**Abstract** High-cost equipment is often reused after maintenance, and whether the information before the maintenance can be used for the Remaining Useful Life (RUL) prediction after the maintenance is directly determined by the consistency of the degradation pattern before and after the maintenance. Aiming at this problem, an RUL prediction method based on the consistency test of a Wiener process is proposed. Firstly, the parameters of the Wiener process estimated by Maximum Likelihood Estimation (MLE) are proved to be biased, and a modified unbiased estimation method is proposed and verified by derivation and simulations. Then, the $h$ statistic is constructed according to the reciprocal of the variation coefficient of the Wiener process, and the sampling distribution is derived. Meanwhile, a universal method for the consistency test is proposed based on the sampling distribution theorem, which is verified by simulation data and classical crack degradation data. Finally, based on the consistency test of the degradation model, a weighted fusion RUL prediction method is presented for the fuel pump of an airplane, and the validity of the presented method is verified by accurate computation results of real data, which provides a theoretical and practical guidance for engineers to predict the RUL of equipment after maintenance.


## 1. Introduction

With the increase of reliability and lifetime, it is difficult to obtain failure data of products in a short term, so the traditional failure data based Remaining Useful Life (RUL) prediction methods are limited in utilization.[1,2] Fortunately, failures of most products are a result of slow degradation of materials, which can be reflected by some performance characteristics gradually.[3] Therefore, reliability and RUL research based on degradation data gains much attention all over the world,

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and becomes the key point of Prognostic and Health Management (PHM).[4]

Degradation modeling is the core problem of RUL prediction. Considering the complex mechanism of a failure, it is difficult to establish a physical failure model for some products with high reliability and long life, while a data-driven statistical method owns obvious advantages.[4,5] A random effects degradation model was applied to describe unit-to-unit variations of test units and studied for highly reliable light displays in Ref. [6], such as Plasma Display Panels (PDPs) and Vacuum Fluorescent Displays (VFDs). The random effects model is a stochastic variable model, which cannot express a dynamic degradation process with time. Therefore, a stochastic process based life prediction method has been developed. The random effects model and the stochastic process model were compared by Peng and Tseng.[7] A stochastic diffusion process was described in detail in Ref. [8], and the Mean-Time-To-Failure (MTTF) of a real Light Emitting Diode (LED) under accelerated testing was achieved. The Gamma process[9] and the inverse Gaussian process[10,11] have been extensively applied to degenerate modeling due to their good statistical properties. The random trajectory model and the stochastic process model have been reviewed in Ref. [12]. Compared to the Gamma process and the inverse Gaussian process, the Wiener process can express the non-monotonicity of the degeneration trend, which has gained great attention. In Ref. [13], a Wiener process with a measurement error was used to study the RUL of a lithium battery. In Ref. [14], the Wiener process was applied to study accelerated step stress degradation. In Ref. [15], the real-time update of RUL prediction was realized by a Wiener process accompanied with recursive filtering. A new class of Wiener process was proposed in Ref. [16], which considers the correlation between the drift parameter and the diffusion parameter, and the proposed Wiener process was demonstrated by a dataset of fatigue crack growth and a dataset of head wears of hard disk drives. Parameters estimation is a part of the major problems in degradation modeling of a Wiener process. Maximum Likelihood Estimation (MLE) is the most widely researched method in the Refs. [17–19]. However, it is difficult to realize an asymptotically unbiased estimation based on MLE, while the number of samples or the number of observations is small, which may lead to large errors of estimated parameters. Therefore, an unbiased parameters estimation method of the Wiener process in a small-sample case is very important. To solve this problem, an unbiased parameters estimation method for the Wiener process based on modified MLE is proposed.

Degradation modeling is often researched under the assumption of mechanism consistency in Refs. [20,21]. Whether the information of different samples or the different stages of the same sample can be fused is based on the consistency of the degradation pattern, so the consistency problem is still ubiquitous in practical engineering. For example, limited by the cost and the actual situation, only a small sample of degradation data can be achieved. A failure happens when degradation reaches a given threshold,[22,23] and the product can continue to work by replacing some components. Whether the degradation data before the maintenance can be used in RUL prediction after the maintenance is based on the consistency of the degradation data before and after the maintenance, which has never been studied before. Aiming at this problem, considering the independent incremental feature of the Wiener process, a Wiener process variation coefficient based consistency test method is proposed, which is not affected by the sampling time and is suitable for small samples.

The remainder of this paper is organized as follows. Section 2 introduces the basic theory of the Wiener process and RUL. In Section 3, a modified unbiased MLE method is proposed based on the statistical properties of the Wiener process, and simulations are studied. A Wiener process variation coefficient based consistency test method is introduced in Section 4, where simulation data and classic crack data are used for validation of the proposed method. In Section 5, the degradation of an airborne fuel pump is studied, and the RUL prediction after maintenance is discussed; while the consistency test is passed, the work provides a theoretical and practical guidance for engineering.

## 2. Basic theory of Wiener process and RUL

Let $\{X(t), t \geqslant 0\}$ be a continuous random process which satisfies $X(0) = 0$, $t$ is time, if any two disjoint time intervals with stationary independent increments $\Delta X(t) = X(t + \Delta t) - X(t)$ obey normal distribution and satisfy

$$\Delta X(t) \sim N(\theta \Delta t, \sigma^2 \Delta t) \tag{1}$$

where $\Delta t$ is the time interval. The Wiener process $\{X(t), t \geqslant 0\}$ is given by

$$X(t) = \theta t + \sigma B(t) \tag{2}$$

where $\theta$ denotes the drift parameter and $\sigma(\sigma > 0)$ the diffusion parameter, while $B(\cdot)$ denotes the standard Brownian motion function.

The degradation process can be described by $\{X(t), t \geqslant 0\}$. $l$ denotes the failure threshold, $\xi$ is defined as the life of the product which is the first time arrived at the failure threshold $l$, then $\xi = \inf\{t | X(t) \geqslant l\}$ is a random variable of inverse Gaussian distribution, and the distribution function $F(\cdot)$ and Probability Density Function (PDF) $f(\cdot)$ are given by[24]

$$F(t; \theta, \sigma, l) = \Phi\left(\frac{\theta t - l}{\sigma \sqrt{t}}\right) + \exp\left(\frac{2\theta l}{\sigma^2}\right) \Phi\left(-\frac{\theta t + l}{\sigma \sqrt{t}}\right) \tag{3}$$

$$f(t; \theta, \sigma, l) = \frac{l}{\sqrt{2\pi\sigma^2 t^3}} \exp\left[-\frac{(l - \theta t)^2}{2\sigma^2 t}\right] \tag{4}$$

where $\Phi(\cdot)$ is standard normal distribution function.

If the degradation data $X(t_k) = x_k$ is obtained, denote the RUL at $t_k$ as $t$, the condition distribution function is shown as

$$\begin{aligned} F_\xi(t | X_1, X_2, \cdots, X_k) &= P(\xi \leqslant t | X_1, X_2, \cdots, X_k) \\ &= P(X(t + t_k) - X(t_k) \geqslant l - X(t_k)) \end{aligned}$$

where $P(\cdot)$ is the probability expressions. As the Wiener process is an independent increment process, the condition distribution function of the RUL can be simplified as[15]

$$F_\xi(t | X_1, X_2, \cdots, X_k) = P(\xi \leqslant t | X_1, X_2, \cdots, X_k) = P(X(t) \geqslant l - X(t_k))$$

Replacing the failure threshold $l$ by $l - x_k$ in Eq. (4), the density function of the RUL can be get as

$$f(t; \theta, \sigma, l) = \frac{l - x_k}{\sqrt{2\pi\sigma^2 t^3}} \exp\left[-\frac{(l - x_k - \theta t)^2}{2\sigma^2 t}\right] \tag{5}$$