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Control method for multi-input multi-output non-Gaussian random vibration test with cross spectra consideration

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Abstract A control method for Multi-Input Multi-Output (MIMO) non-Gaussian random vibration test with cross spectra consideration is proposed in the paper. The aim of the proposed control method is to replicate the specified references composed of auto spectral densities, cross spectral densities and kurtoses on the test article in the laboratory. It is found that the cross spectral densities will bring intractable coupling problems and induce difficulty for the control of the multi-output kurtoses. Hence, a sequential phase modification method is put forward to solve the coupling problems in multi-input multi-output non-Gaussian random vibration test. To achieve the specified responses, an improved zero memory nonlinear transformation is utilized first to modify the Fourier phases of the signals with sequential phase modification method to obtain one frame reference response signals which satisfy the reference spectra and reference kurtoses. Then, an inverse system method is used in frequency domain to obtain the continuous stationary drive signals. At the same time, the matrix power control algorithm is utilized to control the spectra and kurtoses of the response signals further. At the end of the paper, a simulation example with a cantilever beam and a vibration shaker test are implemented and the results support the proposed method very well.

1. Introduction

The traditional Multi-Input Multi-Output (MIMO) random vibration test is mainly to force the multiple outputs to have the specified reference power spectra and the test can only be used for the case of stationary Gaussian random vibration. However, non-Gaussian random vibration environments, such as the action of atmospheric turbulence on aircraft, the acoustic excitation by reaction engine and the vibration by

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combustion instability, are often encountered in aerospace engineering. It is important to monitor the dynamic behavior of the aerospace structures in these non-Gaussian vibration environments. Furthermore, a structure exposure of the same spectra by the Gaussian vibrations or non-Gaussian vibrations will have different damages.[1–4] Hence, it is necessary to research the method for MIMO non-Gaussian vibration environmental test.

In recent years, method for the simulation of non-Gaussian random signal has become one of the important topics in many fields, especially in the simulation of wind forces, radar clutters, sea waves and road vehicle vibrations.[5–10] Some methods are used such as Zero Memory NonLinear (ZMNL) transformation, Auto Regressive Moving Average (ARMA) models, filtered Poisson process, phase modification, alpha stable process, and spherically invariant random vectors process. In the random vibration environmental test, the ZMNL transformation and phase modification are most widely applied.[11–13]

ZMNL transformation method is based on ZMNL monotonic functions, among which the most classical one is Hermitian polynomial. Winterstein developed a Hermitian moment model to transform a Gaussian process into a non-Gaussian process.[14] But this method has some inherent shortcomings and some modified forms are suggested by other scholars later.[15] Smallwood presented three kinds of the ZMNL functions, and each covers similar but slightly different ranges of skewness and kurtosis.[11] The ZMNL transformation method is simple and computationally efficient, but it may induce harmonic distortion and significant dynamic range loss.[11,16]

Phase modification is also a commonly used method to generate non-Gaussian random vibration signals. It is noted that the Auto Spectral Density (ASD) of a random vibration signal is only related to the amplitudes of its Fourier spectrum; hence the kurtosis of the signal can be adjusted by modifying the phase angles without changing its ASD. Steinwolf gave an analytic phase modification formula to generate a non-Gaussian signal with a specified kurtosis from a Gaussian signal.[17,18] Smallwood utilized a non-uniform phase distribution method to realize the non-Gaussian signal with specified skewness and kurtosis and only kurtosis greater than or equal to 3 can be produced by the method.[19] Seong and Peterka constructed the Fourier phases by using the four parameterized phase angles.[20] Hsueh and Hamernik set the Fourier phase to zero within the selected band of frequencies to synthesize the non-Gaussian signal.[21] Generally, phase modification method is a good technique to generate non-Gaussian random signal, but its computational efficiency is not very well if the specified kurtosis is large.

Single Input Single Output (SISO) random vibration test has been widely performed in the laboratory for tens of years. But it is recognized that SISO test is inadequate to simulate the multi-dimensional vibration environments in the real fields.[22,23] MIMO random vibration test has been emerging and applied along with the advancement of hardware and software. Compared to SISO test, it is much difficult to generate the drive signals in MIMO test. Smallwood and Paez contributed to some methods for the generation of stationary Gaussian random drive signals for MIMO test.[24] But the methods are difficult to be extended to the MIMO non-Gaussian case.[25,26] The spectra and kurtoses of the responses should be controlled simultaneously in an MIMO non-Gaussian random vibration test. The kurtoses are used to measure the amplitude distribution characteristics of the responses in time domain and the spectra are used to represent the vibration intensity in frequency domain. Note that the signal in random vibration test is always set to be zero-mean and zero-skewness, so in this paper we only use kurtosis to describe the non-Gaussian characteristic of a random signal.

In some circumstances, the reference spectra are only defined as a diagonal matrix of auto spectral densities in an MIMO random vibration test.[22] In such cases, MIMO random vibration test becomes relatively simple, because only the auto spectral densities need to be controlled and the intractable coupling problems induced by cross spectral densities need not to be considered.[26] But, the cross spectral densities are very important and they determine the phase and coherence relationships among the outputs. So, the cross spectral densities should also be controlled in order to simulate the vibration environments more realistically. However, in MIMO case, the control to non-Gaussian random vibration test will become very difficult if the cross spectral densities are taken into account. Thus, the authors aim to solve this problem in the paper.

2. Generation of non-Gaussian random signal

It is known that a Gaussian signal and a non-Gaussian signal can have the same ASD but different kurtoses. As shown in Fig. 1, three random signals have the same ASD but different kurtoses.

Zero-mean stationary Gaussian random signal can be completely determined by its standard deviation. But for zero-mean zero-skewness stationary non-Gaussian signal, kurtosis must also be taken into consideration. Normalized kurtosis is defined as the fourth statistical moment divided by the square of the second statistical moment as

$$K = \frac{E[x^4(t)]}{E^2[x^2(t)]} \tag{1}$$

where $x(t)$ is a random signal. With this definition, the kurtosis of a Gaussian signal is equal to 3 and the kurtosis of a non-Gaussian signal is not equal to 3. Random signal with a kurtosis greater than 3 is said to be leptokurtic or super-Gaussian and random signal with a kurtosis less than 3 is said to be platykurtic or sub-Gaussian. Because moment higher than the fourth is difficult to estimate, kurtosis is always the only parameter used to measure the non-Gaussian characteristic of a random signal in the engineering practice.

As mentioned above, a non-Gaussian signal can be generated from a Gaussian signal by the ZMNL transformation method. Here, we suggest an improved ZMNL transformation method in order to overcome the defects from the original one. The improved ZMNL transformation method is based on a ZMNL function as

$$g(x) = \begin{cases} \mathrm{e}^{ax} - \mathrm{e}^{-bx} & K > 3 \\ \lg\left(ax + \sqrt{(bx)^2 + 1}\right) & 0 < K \le 3 \end{cases} \tag{2}$$

where $g(x)$ is the resulted non-Gaussian signal and x is the Gaussian signal. The constants a and b are selected to control the skewness and kurtosis of $g(x)$. When $a = b$, skewness is equal to zero. K represents the kurtosis range of $g(x)$. This improved method should be performed in an iterative process