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Nonlinear response analysis and experimental verification for thin-walled plates to thermal-acoustic loads

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Abstract For large deflection strongly nonlinear response problem of thin-walled structure to thermal-acoustic load, thermal-acoustic excitation test and corresponding simulation analysis for clamped metallic thin-walled plate have been implemented. Comparing calculated values with experimental values shows the consistency and verifies the effectiveness of calculation method and model for thin-walled plate subjected to thermal-acoustic load. Then this paper further completes dynamic response calculation for the cross reinforcement plate under different thermal-acoustic load combinations. Based on the obtained time-domain displacement response, analyses about structure vibration forms are mainly focused on three typical motions of post-buckled plate, indicating that the relative strength between thermal load and acoustic load determines jump forms of plate. The Probability spectrum Density Functions (PDF) of displacement response were drawn and analyzed by employing statistical analysis method, and it clearly shows that the PDF of post-buckled plate exhibits bimodal phenomena. Then the Power Spectral Density (PSD) functions were used to analyze variations of response frequencies and corresponding peaks with the increase of temperatures, as well as how softening and hardening areas of the plate are determined. In the last section, this paper discusses the change laws of tensile stress and compressive stress in pre/post buckling areas, and gives the reasons for *N* glyph trend of the stress Root Mean Square (RMS).

1. Introduction

The skin of hypersonic vehicle, the jet nozzle of ramjet engine, the flame tube of aircraft engine, afterburner liner, etc. will encounter severe work environment due to the joint actions of aerodynamic load, thermal load, acoustic load and mechanical load, where the surface temperature of the aircraft Thermal Protection System (TPS) may reach to 1648 °C, and the

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Sound Pressure Level (SPL)may reach to 180 dB.[1]Thin-walled structure used to improve the aerospace craft flight performance will exhibit a complex nonlinear response[2–6], including linear random vibration around the pre-buckled initial equilibrium position, snap-through motion around two post-buckled equilibrium positions and nonlinear vibration around one post-buckled equilibrium position, which lead to the premature fatigue failure of thin-walled structure. Therefore, in order to meet the structure design requirements of aerospace thin-walled structure, the preparatory analysis for thermal-acoustic dynamic response becomes the key of the current works.

The simulations of thin-walled structure play an important role in improving the effectiveness and reliability of the tests. Currently, the numerical simulation methods adopted to solve nonlinear response problem mainly contain: perturbation method, Fokker Planck Kolmogorov (FPK) equation, Von Karman-Herrmann large deflection plate equation, Equivalent Linearization (EL) theory, Reduced Order Method (ROM), Galerkin theory and Finite Element Method (FEM). The EL method has already been used to calculate the stress and strain response of the thermal buckling of plate.[7–9] Combining Galerkin method with Monte Carlo method, Vaicaitis and Kavallieratos studied the nonlinear response of metal and composite structure subjected to random excitation.[10,11] Mei et al. used FEM to calculate nonlinear random response of the shell structure to thermal-acoustic excitation.[12]

Meanwhile, considering the thermal-acoustic nonlinear response problems of aerospace thin-walled structure, NASA Langley research center and the US Air Force Wright-Patterson Flight Dynamics Laboratory (AFFDL) focused on the response characteristics of the thin-walled structure to thermal-acoustic load, and further completed the thermal-acoustic test of aluminum plate in progressive wave tube.[13] Then Rizzi expounded the test methods for dynamic strain and sonic fatigue of thin-walled structure in thermal-acoustic environment.[14] Based on the single mode equation, Ng conducted the nonlinear response analysis tests by using clamped metallic plate and composite plate and obtained the response characteristics.[15] Jacobson performed thermal-acoustic tests to assess the suitable composite panels for ASTOV.[16] Through using high-temperature random fatigue equipment and high-temperature progressive wave tube, Jacobs et al. of McDonnell Douglas studied fatigue performance of ceramic matrix composites.[17] In recent years, NASA carried out a series of structure thermal tests and thermal-acoustic projects, which are based on the flight environments of reusable flight vehicles X-33, X-37 and hypersonic validation machine X-43A.

Based on Ref. 6, this paper selects a new-type material of aircraft engine, and conducts the acoustic vibration test of the clamped plate in thermal environment, obtaining the results of acceleration response and strain response. Then, based on the Von Karman large deflection theory, the FEM/ROM method is used to calculate dynamic response of the plate. Through comparing the response results between simulation and test, the results show the consistency, which verifies the effectiveness of thermal-acoustic calculation method. Finally, the calculation model is used to calculate thermal-acoustic response of the clamped cross reinforced metal plate, and the detailed analysis is performed.

2. Nonlinear response analysis

2.1. Large deformation nonlinear equation for thin-walled structure

Based on Von Karman large deflection plate theory[18] and Kirchhoff's assumptions, the strain equation of a point which has the arbitrary distance from mid-plane is:

$$\begin{cases} \varepsilon_x^{(z)} = \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 - z\dfrac{\partial^2 w}{\partial x^2} \\ \varepsilon_y^{(z)} = \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 - z\dfrac{\partial^2 w}{\partial y^2} \\ \gamma_{xy}^{(z)} = \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\cdot\dfrac{\partial w}{\partial y} - 2z\dfrac{\partial^2 w}{\partial x \partial y} \end{cases} \tag{1}$$

where u and v are x and y direction displacements in mid-plane respectively, and w is lateral deflection of z direction.

The strain compatibility equation which is obtained by performing differential operation between strain and stress is:

$$\frac{\partial^2 \varepsilon_x^{(0)}}{\partial y^2} + \frac{\partial^2 \varepsilon_y^{(0)}}{\partial x^2} - \frac{\partial^2 \gamma_{xy}^{(0)}}{\partial x \partial y} = \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2} \tag{2}$$

where $\varepsilon_x^{(0)}$, $\varepsilon_y^{(0)}$ and $\gamma_{xy}^{(0)}$ are strain components. And by introducing the Airy stress function[19] F, the membrane stresses including $\sigma_x^{(0)}$, $\sigma_y^{(0)}$ and $\tau_{xy}^{(0)}$ can be derived:

$$\begin{cases} \sigma_x^{(0)} = \dfrac{N_x}{h} = \dfrac{1}{h}\cdot\dfrac{\partial^2 F}{\partial y^2} \\ \sigma_y^{(0)} = \dfrac{N_y}{h} = \dfrac{1}{h}\cdot\dfrac{\partial^2 F}{\partial x^2} \\ \tau_{xy}^{(0)} = \dfrac{N_{xy}}{h} = -\dfrac{1}{h}\cdot\dfrac{\partial^2 F}{\partial x \partial y} \end{cases} \tag{3}$$

where N_x, N_y and N_{xy} denote the membrane force, and h is the thickness of plate.

Taking Eq. (3) into Eq. (2), the strain compatibility equation[20] presented by stress function is:

$$\nabla^4 F + Eh\alpha\nabla^2\overline{T} = Eh\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2}\right] \tag{4}$$

where $F = F_h + F_p$ is composed of harmonic solution F_h and particular solution F_p; E is modulus of elasticity; α is coefficient of thermal expansion; $\overline{T}$ is the average temperature along the thickness direction, ∇^2 is Laplace operator and ∇^4 is biharmonic operator. By assuming that temperature is linearly distributed along the thickness, the temperature is:

$$T(x,y,z) = \frac{1}{h}\int_{-h/2}^{h/2} T(z)\mathrm{d}z + z\theta(x,y) = \bar{T}(x,y) + z\theta(x,y) \tag{5}$$

where $T(x,y,z)$ denotes the temperature function of plate, and θ is the temperature gradient along the thickness of plate.

Considering damping force, acoustic load, inertia force, the stress and corresponding shear force, membrane force, and bending moment comprehensively, the stress analysis of plate is carried out to obtain Von Karman large deflection motion equation with temperature.

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