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Boundary discontinuous Fourier analysis of thick beams with clamped and simply supported edges via CUF

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Abstract This paper presents an analytical solution for static analysis of thick rectangular beams with different boundary conditions. Carrera's Unified Formulation (CUF) is used in order to consider shear deformation theories of arbitrary order. The novelty of the present work is that a boundary discontinuous Fourier approach is used to consider clamped boundary conditions in the analytical solution, unlike Navier-type solutions which are restricted to simply supported beams. Governing equations are obtained by employing the principle of virtual work. The numerical accuracy of results is ascertained by studying the convergence of the solution and comparing the results to those of a 3D finite element solution. Beams subjected to bending due to a uniform pressure load and subjected to torsion due to opposite linear forces are considered. Overall, accurate results close to those of 3D finite element solutions are obtained, which can be used to validate finite element results or other approximate methods.

1. Introduction

1D theories are widely used to analyze behaviors of slender bodies in a computationally efficient manner. For this reason, many beam models have been developed. The most well-known beam theory is the classical or Euler-Bernoulli beam theory, which yields reasonably good results for slender beams. However, this model does not take into account shear deformations in a beam. The Timoshenko beam theory is an improvement over the classical theory that considers a uniform shear distribution across the thickness of a beam. However, this theory requires a shear correction factor to correct the strain energy of deformation. Discussion of shear coefficients has been presented in Refs.[1–4]

A large amount of Higher-order Shear Deformation Theories (HSDTs) have been developed in order to consider a nonuniform shear distribution in a beam's cross-section.

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HSDTs with polynomial distributions of shear deformation across the thickness are common due to their simplicity, and some have been presented in Refs.[5–13] Theories containing trigonometric functions in thickness coordinates are also common. A trigonometric shear deformation theory has been presented by Dahake and Ghugal.[14] Many polynomial and trigonometric deformation theories have been developed for analysis of laminated beams, as presented in Refs.[15–20] First-order shear deformation theories are popular due to their computational efficiency, and some have been given in Refs.[21–24]

In order to analyze theories with arbitrary order in a systematic manner, a unified formulation known as Carrera's Unified Formulation (CUF) has been developed in Ref.[25] This formulation has been applied to solve multifield problems, as presented in Refs.[26–28] Carrera and Giunta[29] used the 1D-CUF model to analyze 1D problems with complex cross-sections, and further development has been presented by Carrera et al.[30–32] The capability of these models to obtain quasi-3D solutions has been exploited to develop accurate static[33], free vibration[34,35], and buckling analysis[36] of composite beams.

Analytical solutions for bending of simply supported beams are obtained by using a Fourier series in Navier-type solutions. Other boundary conditions such as clamped conditions can be considered in a finite element formulation or by using the Ritz method, but accurate analytical solutions for these boundary conditions are a fairly scarce topic in the literature. Since finite element formulations or variational methods obtain approximate results, exact analytical solutions are required as a benchmark in order to assess the validity of the results. The present work intends to provide such analytical solutions for clamped boundary conditions.

A generalization of the Fourier series method known as the boundary discontinuous Fourier method can take into account clamped boundary conditions. This method was developed by Chaudhuri in Refs.[37,38] Discontinuities are introduced in order to satisfy boundary constraints. This solution methodology has been applied for static and free vibration analysis of cylindrical panels,[39,40] doubly-curved panels,[41–48] and plates.[49–54] Since the rate of convergence of a Fourier series is slower in the presence of discontinuities, a mixed Fourier solution has also been developed in Refs.[55,56] in order to produce accelerated convergence. Oktem and Chaudhuri have applied the boundary discontinuous Fourier method for analysis of plates[57–59] and shells[60–63] using HSDTs.

In this paper, an analytical solution for static analysis of thick beams with clamped-clamped (C-C) and clamped-simple (C-S) boundary conditions is obtained. A general approach to obtain such an analytical solution using a unified formulation is currently unavailable in the literature, since the other option commonly used for static analysis of beams is a Navier-type solution, which can only consider simply supported edges. Theories of arbitrary order are considered in a systematic manner by using CUF. The principle of virtual work is used to obtain governing equations. The convergence of the solution is analyzed and 3D finite element solutions are obtained in order to assess the validity of results. Good results agreements with 3D finite element solutions are obtained. The results can be used as a benchmark for comparison with approximate solution methods.

2. Analytical modeling

A beam of length L, width b, and total thickness h is considered in the present analysis. The rectangular Cartesian coordinate system used in the present work is shown in Fig. 1. The beam occupies the following region: $-b/2 \leqslant x \leqslant b/2$, $0 \leqslant y \leqslant L$, $-h/2 \leqslant z \leqslant h/2$.

2.1. Elastic stress-strain relations

A general displacement vector is introduced:

$$\boldsymbol{u}(x,y,z) = [u_x \quad u_y \quad u_z]^{\mathrm{T}} \tag{1}$$

The cross-sectional plane of the beam is denoted by Ω. The stress and strain components are grouped as

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{p}} = [\sigma_{zz} \quad \sigma_{xx} \quad \sigma_{zx}]^{\mathrm{T}} \\ \boldsymbol{\varepsilon}_{\mathrm{p}} = [\varepsilon_{zz} \quad \varepsilon_{xx} \quad \varepsilon_{zx}]^{\mathrm{T}} \\ \boldsymbol{\sigma}_{\mathrm{n}} = [\sigma_{zy} \quad \sigma_{xy} \quad \sigma_{yy}]^{\mathrm{T}} \\ \boldsymbol{\varepsilon}_{\mathrm{n}} = [\varepsilon_{zy} \quad \varepsilon_{xy} \quad \varepsilon_{yy}]^{\mathrm{T}} \end{cases} \tag{2}$$

where σ_{ij} and ε_{ij} are the components of the stress and strain vectors, respectively. Subscript "p" stands for terms lying on planes orthogonal to the cross-section, while subscript "n" stands for terms lying on the cross-section. Considering small amplitude displacements, the strain-displacement relations are

$$\begin{cases} \boldsymbol{\varepsilon}_{\mathrm{p}} = \boldsymbol{D}_{\mathrm{p}}\boldsymbol{u} \\ \boldsymbol{\varepsilon}_{\mathrm{n}} = \boldsymbol{D}_{\mathrm{n}}\boldsymbol{u} = (\boldsymbol{D}_{\mathrm{n}\Omega} + \boldsymbol{D}_{\mathrm{n}y})\boldsymbol{u} \end{cases} \tag{3}$$

The linear differential operators $\boldsymbol{D}_{\mathrm{p}}$, $\boldsymbol{D}_{\mathrm{n}\Omega}$, and $\boldsymbol{D}_{\mathrm{n}y}$ are given by

$$\begin{cases} \boldsymbol{D}_{\mathrm{p}} = \begin{bmatrix} 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & 0 & 0 \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix} \\ \boldsymbol{D}_{\mathrm{n}\Omega} = \begin{bmatrix} 0 & \frac{\partial}{\partial z} & 0 \\ 0 & \frac{\partial}{\partial x} & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \boldsymbol{D}_{\mathrm{n}y} = \begin{bmatrix} 0 & 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \end{bmatrix} \end{cases} \tag{4}$$

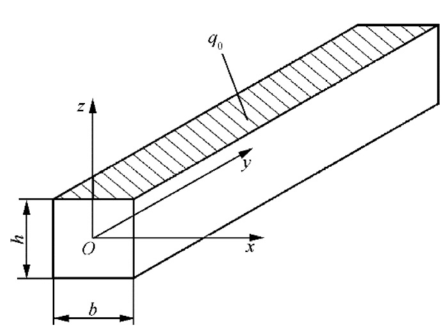

Fig. 1 Coordinate frame of beam model.

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