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Linear stability analysis of interactions between mixing layer and boundary layer flows

Fengjun Liu [a,*], Yifan Wang [b], Ying Piao [b]

[a] *Science and Technology on Scramjet Laboratory, Beijing Power Machinery Institute, Beijing 100074, China*
[b] *School of Aerospace Engineering, Tsinghua University, Beijing 100084, China*

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Abstract The linear instabilities of incompressible confluent mixing layer and boundary layer were analyzed. The mixing layers include wake, shear layer and their combination. The mean velocity profile of confluent flow is taken as a superposition of a hyperbolic and exponential function to model a mixing layer and the Blasius similarity solution for a flat plate boundary layer. The stability equation of confluent flow was solved by using the global numerical method. The unstable modes associated with both the mixing and boundary layers were identified. They are the boundary layer mode, mixing layer mode 1 (nearly symmetrical mode) and mode 2 (nearly anti-symmetrical mode). The interactions between the mixing layer stability and the boundary layer stability were examined. As the mixing layer approaches the boundary layer, the neutral curves of the boundary layer mode move to the upper left, the resulting critical Reynolds number decreases, and the growth rate of the most unstable mode increases. The wall tends to stabilize the mixing layer modes at low frequency. In addition, the mode switching behavior of the relative level of the spatial growth rate between the mixing layer mode 1 and mode 2 with the velocity ratio is found to occur at low frequency.

1. Introduction

Linear stability of the flat plate boundary layer flow is a classical flow problem, which has been fully studied by researchers.[1] The numerical solution obtained via solving the Orr-Sommerfeld stability equation is useful and reliable[1–3] in describing the characteristics of an initial stage of boundary layer transition. The neutral curve of the flat plate boundary layer describes the stable area and the unstable area in the coordinate system formed by Reynolds number and frequency, showing the instability characteristics viewed from a linear perspective. Besides, the linear stability results can be utilized as initial-boundary conditions for direct numerical simulation or large eddy simulation in order to further study flow evolving process downstream. For instance, by forcing T-S disturbance waves solved from linear stability calculations at the inlet of a flat plate boundary layer, Joslin et al.[4,5] performed numerical studies on the boundary layer transition induced by

* Corresponding author.
E-mail address: liu8024@126.com (F. Liu).

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subharmonic waves and got the downstream nonlinear evolving characteristics of the subharmonics and their higher harmonics. The simulated flow transition agrees well with experimental results. Recently, some numerical transition prediction models are developed to study boundary layer transition phenomenon.[6,7]

Flow phenomena of mixing layers, including shear layer and wake, are very common in various fluid machineries. Same as the flat plate boundary layer flow stability problem, both shear layer and wake flow stabilities have been studied extensively and systematically.[8–11] For a wake flow, there exist two unstable modes[8,9] which are symmetric and anti-symmetric respectively. Of these two modes, the growth rate of the symmetric mode is higher, which has been confirmed by experiments.[9] For a shear layer flow, there is only one unstable mode at a given disturbance frequency.[10,11] Taking a hyperbolic tangent function as the mean flow velocity profile for a shear layer, Michalke[10] studied the spatial growth characteristics of disturbance for an incompressible inviscid shear layer, and showed the variations of eigenvalues and eigenfunctions with disturbance frequency. Koochesfahani and Frieler[11] analyzed the linear stabilities of the mixing layer consisting of a wake and a shear layer. In their investigations, a hyperbolic tangent function and an exponential function are applied for the shear layer mean flow and the wake mean flow, respectively. The density differences of two streams are taken into consideration. If the high-density fluid is located on the low-speed stream side, both the unstable shear layer mode and wake mode have almost the same growth rate. Otherwise the instability characteristics are similar to the mixing layer flow with uniform stream density, in which the shear layer mode is dominant.

The confluent wake and boundary layers are often observed in the multi-element airfoil with high lift. When an aircraft takes off or lands, the slat and the flap of its airfoil move forward and backward respectively to increase lift, forming a multi-element airfoil system. When the wake generated by the slat develops downstream, it will locate above the succeeding element, or main element, and the confluent wake and boundary layer flow occur over a significant portion of the main element. The main element wake may interact with the boundary layer of the succeeding flap so as to form a confluent wake and boundary flow, too. Experiments[12,13] show that the confluent wake and boundary layer can exhibit entirely different behavior depending on the operation conditions such as Reynolds number and the relative position among the airfoil elements.

Except for the multi-element airfoil, the confluent mixing and boundary layers can be observed in the combustor of scramjet engines. In the duel-mode scramjet combustor, the strut[14,15] with fuel injection is usually employed to enhance fuel/air mixing. The shear layer at the strut trailing edge interacts with the boundary layer near the combustor chamber wall, and consequently the confluent shear layer and boundary layer flow appears. For the dual combustor ramjet[16–19], the fuel-rich gas from the gas generator mixes with the supersonic air flow from the inlet, and the resulting mixing layer which contains a wake and a shear layer is restricted by the main combustor chamber wall, which leads to the confluent reacting mixing and boundary layer flow. In such a confluent flow, the combusting behavior of the mixing layer may be dramatically different from that of the mixing layer without the restriction of the chamber wall.

One author of this paper conducted the instability analysis for the incompressible and compressible confluent wake and boundary layers.[20,21] For the incompressible confluent flow, one instability mode associated with the boundary layer and the other instability mode associated with the wake were identified. The wake changes the shape of the boundary layer neutral curve, and the resulting critical Reynolds number decreases. In addition, the wake leads to the instability mode associated with the boundary layer to be further unstable, and this instability tends to be strengthened with the velocity defect of the wake.[20] Therefore, the wake plays an important role in the linear development process of the boundary layer disturbance wave and the initial stage of flow transition. At high speeds, in contrast to the incompressible results, a reduced position between the wake and the boundary layer has a strong stabilizing effect on the growth rates of both the first and the second modes associated with the boundary layer.[21]

It should be noted that under the practical engineering application the initial mean velocity profile for a mixing layer almost has not only a wake component due to the boundary layers on the two sides of the splitter plate, but also a shear layer component due to the velocity difference of the two streams. Such a mixing layer containing a wake and a shear layer interacts with a boundary layer and forms the confluent mixing and boundary layer flow, which is taken into consideration in the present paper. The linear stability theory is used to study the instability of the confluent flow, with the focus on the flow stability interactions between the mixing layer and the boundary layer. The confluent mean flow model is the direct superposition of the Blasius boundary layer flow and the mixing layer flow, where a hyperbolic tangent function is chosen for the shear layer and an exponential function is chosen for the wake. The disturbance amplitude of the confluent flow is assumed to be small enough so that it can be described by the Orr-Sommerfeld stability equation. The central differential scheme with the sixth order accuracy is used to discretize the stability equation and the resulting discretization equation is solved using the global numerical method. The whole eigenvalue spectrum including the discrete and continuous part is obtained. The instability modes associated with the boundary layer and the mixing layer are identified. The effects of the critical gasdynamic and geometric parameters on those instability modes are analyzed and the instability characteristics of the confluent flow are obtained.

The confluent mixing and boundary layer flow model is given in the following section and then the Orr–Sommerfeld equation is briefly derived. It is followed by the global numerical method, discussion for the confluent flow stability analyses and concluding remarks.

2. Confluent flow model

The confluent flow considered here consists of a flat plate boundary layer and a mixing layer which is located a certain distance away from the flat plate. The mean velocity profile of the confluent flow is the superposition of that of the boundary layer and that of the mixing layer.

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