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On solving multi-commodity flow problems: An experimental evaluation

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Abstract Multi-commodity flow problems (MCFs) can be found in many areas, such as transportation, communication, and logistics. Therefore, such problems have been studied by a multitude of researchers, and a variety of methods have been proposed for solving it. However, most researchers only discuss the properties of different models and algorithms without taking into account the impacts of actual implementation. In fact, the true performance of a method may differ greatly across various implementations. In this paper, several popular optimization solvers for implementations of column generation and Lagrangian relaxation are discussed. In order to test scalability and optimality, three groups of networks with different structures are used as case studies. Results show that column generation outperforms Lagrangian relaxation in most instances, but the latter is better suited to networks with a large number of commodities.

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1. Introduction

The multi-commodity flow problem (MCFP) deals with the assignment of commodity flows from source to destination in a network. MCFPs are highly relevant in several fields including transportation[1] and telecommunications.[2]

MCFPs have been studied by a number of researchers for several decades, and a variety of solutions have been proposed such as column generation, Lagrangian relaxation, branch-and-bound, and Dantzig-Wolfe decomposition. Tomlin[3] presented a column generation algorithm first, and he is also one of forerunners of using Dantzig-Wolfe decomposition. Based on these methods, a number of new processes were proposed more recently. For instance, Barnhart et al. presented a partitioning solution approach in order to solve large-scale MCFPs with large numbers of commodities.[4] Many constraints can be relaxed with a cycle-based formulation and column generation. Then, by solving a series of linear programs with reduced size, an optimal solution can be obtained within a finite number of steps. Based on column generation, Barnhart et al. presented a modified version of the branch-and-bound algorithm for solving origin-destination integer multi-

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commodity flow problems.[5] An algorithm for path-based models was improved by presenting a new branching rule and adding cuts. Computational complexity can be reduced significantly by these cuts in many real-world situations. The classical column generation algorithm has a variety of advantages for solving MCFPs. However, because the restricted master problem (RMP) changes a lot with continuous column generation iterations, the solution often convergences slowly. Therefore, Gondzio et al. proposed the primal-dual column generation method (PDCGM) for solving this problem.[6–8] PDCGM is a modified method relying on the sub-optimal dual solutions of restricted master problems where solutions are obtained with the primal-dual interior-point method. This method was initially developed for solving large-scale convex optimization problems. PDCGM was shown by the authors to be competitive and suitable for a wide context of optimization problems.

In addition to column generation, new methods based on Dantzig-Wolfe decomposition and Lagrangian relaxation have been developed. Based on Dantzig-Wolfe decomposition, Karakostas proposed polynomial approximation approaches in order to solve MCFPs.[9] These approaches were also derived from other previous algorithms.[10,11] These methods minimize computation time dependence on the number of commodities. Based on Lagrangian relaxation, Retvari et al. proposed a new Lagrangian relaxation method that can solve the MCFP as a sequence of single-commodity flow problems.[12] This technique performs best when solving OSPF (open shortest path first) traffic engineering problems because a given path can be improved towards approximate optimality while giving several necessary parameters. In addition, Babonneau and Vial proposed a new method based on partial Lagrangian relaxation,[13] which constrains relaxation to the set of arcs that are saturated at the optimum. This method can be used to solve large problems.

In addition to the previous research, modified versions of MCFPs are also studied. Moradi et al. proposed a new column generation method for solving bi-objective MCFP problems.[14] Based on the simplex method and Dantzig-Wolfe decomposition, the algorithm moves between different points. Similar to Karakostas's method, it demonstrates that the average computation time does not necessarily depend on the number of commodities. Path-based models are often used to solve MCFPs; However, Bauguion et al. proposed a new idea about such models.[15,16] Instead of generating paths for each commodity, they generated groups of paths in other ways, such as combining them into single sets or separate trees. These methods reduce computation time compared with other models.

MCFPs can be applied to many different application problems.[17] Zhang et al. presented a multi-commodity model for supply chain networks,[18] which was solved using the Benders decomposition method. Caimi et al. studied the problems of conflict-free train routing and scheduling and proposed a new resource-constrained model based on the multi-commodity flow.[19] Morabito et al. studied network routing for generalized queuing networks and presented a multi-commodity flow algorithm based on a routing step and an approximate decomposition step.[20] Shitrit et al. applied the multi-commodity flow problem to tracking of multiple people. Experimental results showed that their approach performs better than other state-of-the-art tracking algorithms.[21]

As our review of related literature shows, the vast majority of the methods proposed for solving MCFPs discussed the properties of different models and algorithms without considering the impact of implementation. For instance, while solving the MCFP with an algorithm, some portions of the problem can be formulated as a linear program. In this case, it is important to select the appropriate implementation type, i.e. linear program solver.[22] This paper introduces the formulations and processes of two commonly-used algorithms for solving MCFPs: column generation and Lagrangian relaxation. In addition to the algorithm theory, implementations for MCFPs are also discussed. Several popular program solvers, such as GNU linear programming kit (GLPK), CVXPY, GUROBI, and SCIPY, are introduced briefly. Past research often performed comparisons with single, outdated competing methods or implementations. This paper promotes the idea of comparing popular methods in order to decide which one is the best for a given problem. In order to test the scalability and optimality of the algorithms and program solvers, three groups of networks (grid, planar, and airport networks) are chosen as case studies. For each group of networks, the optimality, computation time, and number of iterations for the algorithms and different program solvers are compared.

Results indicate that column generation has better properties for solving an MCFP than Lagrangian relaxation in most instances. However, Lagrangian relaxation can be faster and within acceptable optimality bounds in certain large-scale networks with a high number of commodities. There is a similar relationship with implementations for column generation: CVXPY performs better than GLPK for solving MCFPs with a large number of commodities by column generation while GLPK is superior for a small-scale MCFP. This work shows the tremendous impact of implementation techniques on computation time and solution quality and lays the foundation for further research on MCFPs.

The remainder of this paper is organized as follows: An introduction and formulation of the multi-commodity flow problem are provided in Section 2. Relevant solution techniques, such as column generation, Lagrangian relaxation, and several program solvers, are introduced in Section 3. In order to compare these techniques, three groups of different datasets are discussed in Section 4. The evaluations performed using these datasets as case studies are presented in Section 5. Conclusions are in Section 6.

2. Multi-commodity flow problem

The MCFP seems like a combination of several single-commodity flow problems. However, because of the interaction between commodities, the complexity of MCFP is much higher than that for solving each single-commodity flow problem independently.[23] In order to solve MCFPs, two necessary constraints must be considered. The first is the travel demand, which means that all the commodities need to be transported to their destinations. The second is the edge capacity constraint. This means that the flow on each edge can not exceed flow capacity. The first constraint is essentially the sum of a set of single-commodity flow problems. However, the second

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