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Autonomous Underwater Vehicle Pursuit of Biological Specimens in the Open Ocean

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Abstract: The pursuit equations in two-dimensional space are examined, and then parametrised in terms of relative velocity and initial range. Several inferences about the behaviour of these equations are drawn. The burst speed of several fish species are tabulated, along with several types of Autonomous Underwater Vehicle. An example pursuit calculation is described.

Keywords: pursuit, fish, evasion, curves of pursuit, marine biology, autonomous underwater vehicle, AUV, strategy, analytic solution, path planning

NOMENCLATURE

$\bar{v}$	sustained speed
$\lambda(f)$	wavelength of sound frequency f in water
a	initial separation
c	speed of sound in water
c_1	constant of integration
e	exponential function
f	frequency of auditory signal
k	speed of quarry
n	ratio of pursuer speed to quarry speed
P	location of AUV (pursuer)
p	dummy variable for first derivative
Q	location of fish (quarry)
r	radius
t	time
v_{max}	burst speed
x	first Cartesian coordinate
y	second Cartesian coordinate
y'	first spatial derivative of y
y''	second spatial derivative of y

1. INTRODUCTION

Some population studies in marine biology require that fish be tracked. This need is exemplified by a research partner, the University of the Azores, whose biologists seek to locate particular specimens which have been tagged with acoustic emitters. The biologists hope the acoustic signals which are emitted can indicate the habitat and/or behaviour of the specimen, shoal, or species. The symbiotic relationship between the present researchers and the biologists is to provide the latter with Autonomous Underwater Vehicle (AUV) tools to achieve their goals. Progress towards the shared goal will be obtained if the mathematics of biological pursuit is clarified. Properly defined, the set of all pursuits includes the stationary quarry.

This paper will be organised as follows: the next section will consist of a literature survey. Curves of pursuit will be explained in §3. Some biological applications of the mathematical tool will be explored in consequence. Recommendations for further study will complete the paper.

2. LITERATURE STUDY

The differential equations of pursuit were developed as a result of World War II, so that Yates, R. C. [1952] §9.5.G devoted scarcely two pages to treatment of the subject. These equations are repeated and developed here in the next section. Standard reference material even today was written by Locke, A. S. [1955]. Section 7.9 of Langer, R. E. [1954] had the pursuer and quarry in opposite corners of the Cartesian system, but his results were similar to those of Yates. Stewart did not teach the problem in his first edition Stewart, J. [1987] but added a treatment similar to that of Langer in his third Stewart, J. [2006] at page 554.

The example to be employed in the next section follows Yates, R. C. [1952] closely because his placement of the origin coincides with the initial location of the pursuer. The terminal v^2/r acceleration problem of military pursuit is of no concern because, as explained by Fig. 1, the pursuit of a specimen differs from its capture. The biologists forbid specimen capture in the present instance. Adler, F. P. [1955] knew of three types of guidance systems: pure pursuit, constant-bearing collision, and proportional navigation. His theory of proportional navigation, as updated with complex coordinates by Becker, K. [1990], is still relevant today in the case of the marine biologist. His reason for the elimination of constant-bearing collision–that the algorithm requires instantaneous adjustments of the line-of-sight–holds true for the present application too. He eliminated for his purposes the first option on two grounds: (a) that the mathematics of the pursuit equations forces the system into a tail-chase scenario (which would be unsuitable for miltary defense); and (b) that the terminal turn rate becomes infinite if the ratio of pursuer to quarry speed exceeds two. Doppler effect logic can be

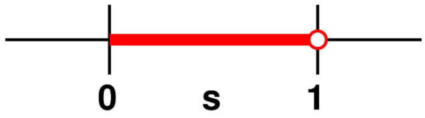

Fig. 1. The strict definition of the problem includes pursuit, marked in red on this numberline, but not capture, as evinced by the domain termination at $s \in [0, 1)$.

employed for biological purposes so that the AUV is only allowed to engage its quarry while the quarry recedes from it, while inspection of Fig. 3 shows his error in case (b) to be of similar type with Zeno's paradox. This study appears to be novel since his ground (and those of his peers) differs from the present ground.

The case of curvature constrained pursuit is of interest, because curvature is the mirror image of centripetal acceleration. Miloh, T. [1982] notes that in certain instances it can be advantageous to both parties to maintain a planar relationship in three-dimensional space, and that the termination of his game-theoretic approach depends substantially on the maximum rate of acceleration of both parties. While the minimum rate of turn of the quarry is unknown and unknowable, the minimum rate of turn of the pursuer is knowable. More recent efforts in this area have focussed on collision avoidance Harris, C. J., *et al.* [1999], Wilson, P. A., C. J. Harris, X. Hong [2003], or incomplete measurements and uncertain systems, eg. Moitié, R., *et al.* [2002] or Shieh, C.-S. [2007]. The latter can be of very limited tractability due to memory requirements Moitié, R., *et al.* [2002] or are tunable Shieh, C.-S. [2007], thus unsuited to automation. But the presumption of the latter that the object of the mathematics is to destroy the quarry is contrary to expectations in the present study. The marine biologist whose aim is observation of the quarry at close range is likely to avoid damage to it, and hence the minimum turn radius is likely to be irrelevant, whereas the military objective, which has informed most previous studies of pursuit, is destruction. Figure 3 demonstrates the need to avoid velocity ratios near to unity because this causes more curvature to be required in the path to intercept.

3. THEORY

The Yates exposition of the pursuit equation, which is repeated here in Fig. 2, differs from the Langer presentation of the same problem in its choice of origin: Yates favours the pursuer, while Langer favours the quarry. The AUV (pursuer, P) and the specimen (quarry, Q) are labelled. The quarry travels in a straight line at maximum speed, while the pursuer acts at all times to minimise its distance to the quarry; that is, P makes no effort to predict the behaviour of Q, and hence it is at all times directed towards Q. The quarry travels Northward at maximum speed k. The pursuer notices Q at time t_o near O, the

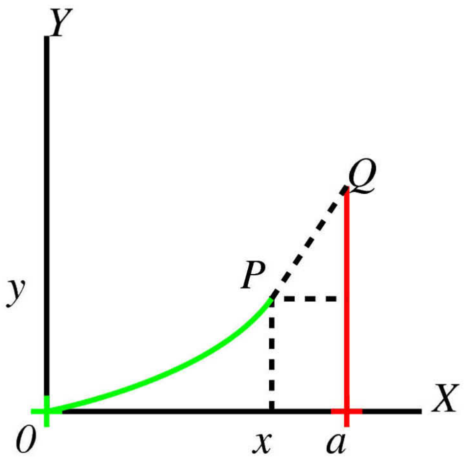

Fig. 2. The problem from the viewpoint of the pursuer, whose path is marked in green. The path of the quarry is marked in red. A cross marks the origin of each.

origin, which is a metres distant from the instant location of Q. Suppose nk to be the speed of P, and r to be the range between the two:

$$nk = \frac{dr}{dt} \tag{1}$$
$$= \frac{\sqrt{dx^2 + dy^2}}{dt} \tag{2}$$
$$= \frac{dx\sqrt{1 + y'^2}}{dt} \tag{3}$$

and the stem of P to be pointed at the location

$$\frac{dy}{dx} = y' = \frac{kt - y}{a - x} \tag{4}$$

Equation 4 completes the physics of the problem, and the mathematics now follows:

$$y'(a - x) = kt - y \tag{5}$$
$$y''(a - x) - y' = k\frac{dt}{dx} - y' \tag{6}$$
$$y''(a - x) - y' = k\frac{\sqrt{1 + y'^2}}{nk} - y' \tag{7}$$
$$ny''(a - x) = \sqrt{1 + y'^2} \tag{8}$$

The result is a nonlinear equation of the second order in which the term y makes no explicit appearance. This allows the substitution $y' = p$ to transform the equation for the path of P to

$$n(a - x)\frac{dp}{dx} = \sqrt{1 + p^2} \tag{9}$$
$$\frac{ndp}{\sqrt{1 + p^2}} = \frac{dx}{a - x} \tag{10}$$