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Extremum Seeking Speed and Heading Control Applied to Parametric Roll Resonance

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Abstract: Parametric roll is a dangerous resonance phenomenon for ships caused by time-varying restoring forces. The resulting large roll oscillations may endanger the ship's safety and possibly lead to capsizing. The susceptibility of container ships to parametric roll resonance has caused considerable research activities in the development of control and operational strategies to avoid large roll motions. In this paper, a novel approach to the active control of ships experiencing parametric roll resonance is presented. The methodology of extremum seeking control is used to iteratively determine the set-point for the wave encounter frequency in order to avoid the conditions for the onset and build-up of parametric roll. The desired trajectory for the wave encounter frequency is tracked by mapping it to the ship speed and heading angle by a control allocation, formulated as a constrained optimization problem, and consequently by implementing speed and heading controllers. The combined variation of the speed and heading angle is shown to efficiently drive the ship out of the parametric roll resonance region.

Keywords: parametric roll resonance; extremum seeking control; control allocation; nonlinear system; ships

1. INTRODUCTION

Parametric roll resonance of ships has attracted considerable interest of the scientific community in the last decades. The increased attention is certainly partly due to the immense importance of the maritime transportation nowadays, which demands a unprecedented growth of the container trade (UNCTAD/RMT/2009, 2009). To meet the demands for faster and more efficient maritime trade, modern container ships are designed featuring a particular hull shape, characterized by a large bow flare and stern overhang, which makes them especially prone to parametric roll resonance (Holden et al., 2007).

Recent incidents of container ships which experienced parametric roll resonance, reported to include significant loss and damage to cargo as well as considerable monetary damages up to millions of dollars (France et al., 2001), caused thorough investigations into the phenomenon of parametric roll resonance of ships.

Parametric roll resonance is a type of nonlinear oscillations which may result in large amplitudes of the ship's roll motion. The oscillation at one of the system's resonant frequencies is due to the time-varying coefficients in the governing differential equation of the roll motion. The parametric excitation is caused by the time-varying geometry of the submerged ship hull and the resultant time-varying restoring forces of the ship. Contrary to externally forced systems, in parametrically excited systems even small excitations can lead to a large response in the system if the excitation frequency is close to twice the system's natural frequency (Nayfeh and Mook, 2004).

For a ship in head sea, four empirical conditions for the onset of parametric roll resonance evolve from the comparison between model tests and the roll motion described by the linear damped Mathieu equation (France et al., 2001):

- The natural period in roll is equal to approximately twice the wave encounter period.
- The wave length is on the order of the ship length.
- The wave height exceeds a critical level.
- The roll damping is low.

The application of active control techniques has the potential to avoid large roll motions of ships effectively but it has attracted the interest of researchers only quite recently. Methods have been investigated which regulate either the roll motion directly by creating an opposing roll moment, or indirectly by violating the empirical criteria for the onset of parametric resonance (Holden et al., 2009; Galeazzi and Blanke, 2007; Galeazzi et al., 2008, 2009).

2. EXTREMUM SEEKING CONTROL

Extremum seeking (ES) is a non-model-based, adaptive control method which iteratively tunes the parameters of an objective function such that the output of it reaches a local extremum. The main drawback of conventional adaptive control methods is that the set-points or the reference trajectories have to been known a priori. ES overcomes this by exciting the plant with a sinusoidal probing signal in order to estimate the gradient of the objective signal online (Ariyur and Krstić, 2003).

ES has been applied to various control problems, amongst others to formation flight, modelling of a bioreactor, and antilock braking (see Ariyur and Krstić, 2003). It has also been used for the online tuning of the parameters of proportional-integral-derivative (PID) controllers (Killingsworth and Krstić, 2006).

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2.1 *Introduction*

Here, the methodology of ES is introduced based on the results presented in Ariyur and Krstić (2003) and Krstić and Wang (2000). An ES control scheme is depicted in Figure 1. It consists of a plant and a dynamic feedback loop (ES loop). The output y has an extremum at $\theta = \theta^*$. The parameter θ parameterizes a smooth control law $u = \alpha(x, \theta)$. The probing signal $a \sin(\omega t)$ adds a slow perturbation to $\hat{\theta}$ which is the best actual estimate of θ^*.

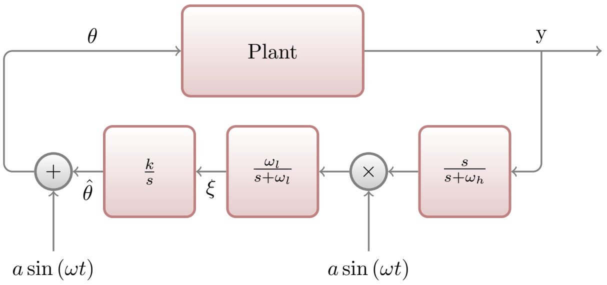

Fig. 1. ES loop for a general plant (Ariyur and Krstić, 2003).

Assuming that the perturbation signal is sufficiently slow compared to the plant dynamics, the plant can be viewed as a static map and its dynamics can be neglected for the ES loop. The high-pass filter $s/(s + \omega_h)$ eliminates the offset of the output y. The second perturbation creates a sinusoidal response of y. Adding a sinusoidal perturbation signal to the best estimate $\hat{\theta}$ will then cause the two sinusoids to be in phase or out of phase depending on whether $\hat{\theta}$ is smaller or bigger than θ^*. The low-pass filter $\omega_l/(s + \omega_l)$ extracts the offset due to the product of the two sinusoids. The approximate gradient update law is given by the integrator in the ES loop.

The ES loop thus drives the parameter θ to θ^* by exciting the plant with a perturbation signal and measuring the gradient information. Then the a priori unknown operating condition, resulting in an extremum of the output of the plant, is met.

Krstić and Wang (2000) define three time scales for ES feedback system:

- **Fastest:** The plant with the controller.
- **Medium:** The perturbation signal.
- **Slowest:** The filters in the the ES loop.

The plant having the fastest time constant, results from the assumption of the static map from reference to output of the plant. The perturbation signal must thus not be faster than the plant or otherwise the exciting sinusoid would not be fed through the plant properly. The filters on the other hand give an estimate of the gradient, which implies that they are on the slowest time scale.

2.2 *Extremum Seeking for Speed and Heading Control*

The frequency condition

$$\omega_e \approx 2\omega_\phi \tag{1}$$

where ω_e is the wave encounter frequency and ω_ϕ the ship's natural roll frequency, provides a possible approach to prevent the ship from experiencing parametric roll resonance. Thus, a rather obvious modus operandi is to avoid the frequency condition while operating a ship or, in case the condition is met, to violate it before any damage is done.

It is assumed that the ship's natural roll frequency is constant. The wave encounter frequency is

$$\omega_e(u, \omega_0, \beta_w) = |\omega_0 - \frac{\omega_0^2}{g} u \cos(\beta_w)| \tag{2}$$

Here, β_w denotes the wave encounter angle which depends on the heading angle ψ, whereas u is the ship speed and ω_0 the modal wave frequency. Both the ship speed and the heading angle are controllable which yields two variables to vary ω_e in order to avoid the frequency condition (1).

The desired trajectories for the ship speed and heading angle depend on natural conditions as well as on the specific operational task of the ship and are a priori not known. Therefore, ES control is applied to iteratively determine the ship's operating point which avoids the frequency condition (1) for the onset of parametric resonance while taking into account operation specific requirements such as the ship's course and speed.

Figure 2 depicts the proposed approach to regulate the roll motion indirectly. The ES loop is applied to determine online the desired wave encounter frequency $\omega_{e,\mathrm{des}}$, which drives the ship out of the dangerous region relevant for the onset of parametric roll resonance. In other words, $\omega_{e,\mathrm{des}}$ is the actual estimate of ω_e^* which produces an extremum of a defined objective function. The desired trajectory $\omega_{e,\mathrm{des}}$ is mapped to the control inputs for the surge and yaw systems by control allocation. The outputs of the surge and the yaw systems, the speed and the heading angle, are input to (2), yielding the wave encounter frequency, which is fed back to the ES loop and also serves as input to the roll system to determine the roll angle.

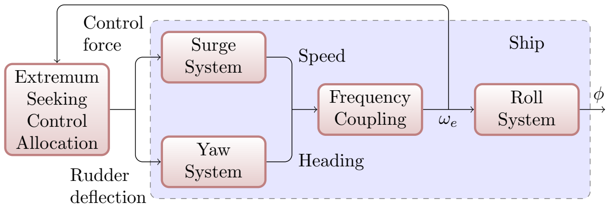

Fig. 2. ES applied to the control of the roll motion of ships.

It is advantageous to design the ES loop with the wave encounter frequency together with a control allocation compared to the multiparameter ES scheme with the two controllable variables ship speed and heading angle (Ariyur and Krstić, 2003). The formulation of a control allocation allows to impose restrictions on the speed and heading variations from their respective nominal values.

2.3 *Ship Model*

The surge and roll dynamics of a ship can be derived from the models presented in Holden et al. (2007) and Fossen (2002). They are given by:

$$\begin{aligned}(I_x + A_{44}(\omega_\phi))\ddot{\phi} - K_p\dot{\phi} - K_{|p|p}|\dot{\phi}|\dot{\phi} + \rho g \nabla \overline{GM}_m \\ + \rho g \nabla \overline{GM}_a \cos(\omega_e t)\phi - K_{\phi\phi\phi}\phi^3 = \tau_\phi\end{aligned} \tag{3a}$$

$$(m + A_{11}(0))\dot{u} - X_u u - X_{|u|u}|u|u = \tau_u \tag{3b}$$

The yaw subsystem is based on the 1^{st}-order Nomoto model (see Fossen, 2002):

$$T\ddot{\psi} = -\dot{\psi} + K\delta \tag{4}$$

The measurements are roll amplitude ϕ, roll rate $\dot{\phi}$, speed u, heading angle ψ and heading rate $\dot{\psi}$.