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Multiaxial fatigue life prediction of composite materials

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Abstract In order to analyze the stress and strain fields in the fibers and the matrix in composite materials, a fiber-scale unit cell model is established and the corresponding periodical boundary conditions are introduced. Assuming matrix cracking as the failure mode of composite materials, an energy-based fatigue damage parameter and a multiaxial fatigue life prediction method are established. This method only needs the material properties of the fibers and the matrix to be known. After the relationship between the fatigue damage parameter and the fatigue life under any arbitrary test condition is established, the multiaxial fatigue life under any other load condition can be predicted. The proposed method has been verified using two different kinds of load forms. One is unidirectional laminates subjected to cyclic off-axis loading, and the other is filament wound composites subjected to cyclic tension-torsion loading. The fatigue lives predicted using the proposed model are in good agreements with the experimental results for both kinds of load forms.

1. Introduction

Due to their high specific strength and high specific stiffness, composite materials have been widely used in many industrial fields, especially in aeronautical and astronautical structures. Taking aircraft engines as examples, some major components fail under multiaxial fatigue loading. Since composite structural components may be subjected to different forms of multiaxial cyclic loading, the task of predicting the multiaxial fatigue lives of structures becomes essential and indispensable.[1]

Multiaxial fatigue of composite materials can be divided into Multiaxial Stress fatigue under Uniaxial Loading (MSUL) and Multiaxial Stress fatigue under Multiaxial Loading (MSML). The multiaxial stress state of MSUL is caused by anisotropy of composite materials, so the ratio of biaxial stress is unchangeable. The multiaxial stress state of MSML is caused by both anisotropy of composite materials and multiaxial loading, so the ratio of biaxial stress can be changed according to external loading.

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The fatigue behavior of composite materials under MSUL has been studied in depth. Theoretical research approaches involve *S-N* equation,[2] models based on residual strength/stiffness,[3,4] and models based on progressive failure mechanisms.[5–7] Under MSML, these theoretical approaches are not suitable because there are numerous combinations of stress components.

For metallic materials, fatigue crack growth is a de-cohesion process along the shear bands of a crack tip. The normal strain on the crack plane accelerates this de-cohesion process.[8] Based on this fatigue damage mechanism, some researchers have proposed different critical plane approaches, such as Findley,[9] Fatemi,[10] Smith,[11] and Shang[12,13] et al. A good correlation with test data has been obtained for different materials and loading conditions. Since a critical plane approach reflects the mechanism of multiaxial fatigue damage, Petermann[14] and Plumtree[15] et al. introduced a critical plane concept into their studies of unidirectional laminates which were subjected to cyclic off-axis fatigue loading. This approach only needs the material properties of fibers and the matrix along with one curve between the fatigue damage parameter and the fatigue life to be known. Therefore, it's very convenient for applications in engineering.

At the mesoscopic level, composite materials consist of fibers and matrix. For simplicity, fibers are always assumed to be distributed periodically in composite materials. Through applying corresponding periodical boundary conditions on an appropriate unit cell, the stress and strain fields in the fibers and the matrix can be analyzed.[16–18]

The objective of this paper is to propose a model for predicting the multiaxial fatigue lives of composite materials which have failed with matrix cracking. The fiber-scale unit cell model is used to calculate the stress and strain fields in the fibers and the matrix. With a newly proposed energy-based fatigue damage parameter, the model can be used to predict the fatigue lives of MSUL and MSML.

2. Method of fatigue life prediction

2.1. Energy-based fatigue damage parameter

The fracture plane of a unidirectional laminate is parallel to the fibers when being subjected to cyclic off-axis loading. With the fracture plane as the critical plane, the critical plane concept was used to deal with the problem of multiaxial fatigue life prediction by Petermann[14] and Plumtree[15] et al. Their fatigue damage parameters are defined as Eqs. (1) and (2), respectively.

$$W^* = \frac{1-R^2}{2}\left(\sigma_{22}^{\max}\varepsilon_{22}^{\max} + \tau_{12}^{\max}\gamma_{12}^{\max}\right) \tag{1}$$

$$W^* = \sigma_{22}^{\max}\Delta\varepsilon_{22} + \tau_{12}^{\max}\Delta\gamma_{12}/2 \tag{2}$$

where R is the ratio of minimum-to-maximum load, σ_{22} and τ_{12} are the normal stress and shear stress on the critical plane, respectively, ε_{22} and γ_{12} are the normal strain and shear strain on the critical plane, respectively, and $\Delta\varepsilon_{22}$ and $\Delta\gamma_{12}$ are the tensile strain range and shear strain range on the critical plane, respectively.

From the perspective of fracture mechanics, there are three basic forms of crack propagation as shown in Fig. 1. For composite materials which have failed with matrix cracking, these three forms of crack propagation are taken into consideration in this paper, and the relevant stress and strain components in the matrix are used to establish a new energy-based fatigue damage parameter in the following part.

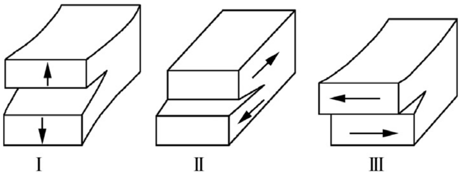

Fig. 1 Basic forms of crack propagation.

Under complex cyclic loading, the stress in the transverse direction of the matrix is not all positive everywhere. Since compressive stress will close the crack and block the propagation of the crack, the damage parameter in the transverse direction of the matrix is given by Eq. (3).

$$\begin{cases} W_{22}^* = \dfrac{1}{2}\left(\sigma_{22}^{\max}\varepsilon_{22}^{\max}u(\sigma_{22}^{\max}) - \sigma_{22}^{\min}\varepsilon_{22}^{\min}u(\sigma_{22}^{\min})\right) \\ W_{22}^* = \dfrac{1}{2}(1-R^2)\sigma_{22}^{\max}\varepsilon_{22}^{\max} \quad \text{when} \quad \sigma_{22}^{\min} \geqslant 0 \end{cases} \tag{3}$$

where $u(x)$ is the unit step function: $u(x) = 1$ when $x > 0$; $u(x) = 0$ when $x < 0$.

Since positive and negative shear stress has the same effect on crack propagation, the damage parameter corresponding to shear stress is given by Eqs. (4) and (5).

$$\begin{cases} W_{23}^* = \dfrac{1}{2}\left(\mathrm{sgn}\left(\tau_{23}^{\max}\right)\tau_{23}^{\max}\gamma_{23}^{\max} - \mathrm{sgn}\left(\tau_{23}^{\min}\right)\tau_{23}^{\min}\gamma_{23}^{\min}\right) \\ W_{23}^* = \dfrac{1}{2}(1-R^2)\tau_{23}^{\max}\gamma_{23}^{\max} \quad \text{when} \quad \tau_{23}^{\min} \geqslant 0 \end{cases} \tag{4}$$

$$\begin{cases} W_{21}^* = \dfrac{1}{2}\left(\mathrm{sgn}\left(\tau_{21}^{\max}\right)\tau_{21}^{\max}\gamma_{21}^{\max} - \mathrm{sgn}\left(\tau_{21}^{\min}\right)\tau_{21}^{\min}\gamma_{21}^{\min}\right) \\ W_{21}^* = \dfrac{1}{2}(1-R^2)\tau_{21}^{\max}\gamma_{21}^{\max} \quad \text{when} \quad \tau_{21}^{\min} \geqslant 0 \end{cases} \tag{5}$$

where sgn(x) is the sign function: sgn(x) = 1 when $x > 0$; sgn(x) = −1 when $x < 0$.

The experimental results of unidirectional laminates show that the *S–N* curve in each material principal direction is not coincident with each other, i.e., each stress component has a different contribution to fatigue damage.

Based on the above analysis, the energy-based fatigue damage parameter of the matrix is defined as

$$W^* = W_{22}^*V_{\mathrm{m}} + \frac{Y^2}{S^2}\left(W_{23}^*V_{\mathrm{m}} + W_{21}^*V_{\mathrm{m}}\right) \tag{6}$$

where V_{m} is the volume of the matrix, Y is the static tensile strength in the transverse direction, and S is the static shear strength in the material principal plane.

When a fiber-scale unit cell is used to analyze the stress and strain fields at the mesoscopic level, the energy-based fatigue damage parameter W^* can be extended as Eq. (7) based on the volume average method.

$$W^* = \frac{1}{V}\sum_{i=1}^{N_{\mathrm{m}}}W_{22i}^*V_i + \frac{1}{V}\cdot\frac{Y^2}{S^2}\sum_{i=1}^{N_{\mathrm{m}}}\left(W_{23i}^*V_i + W_{21i}^*V_i\right) \tag{7}$$

where N_{m} is the number of matrix elements, V_i is the volume of the ith matrix element, and V is the volume of the whole unit cell.

When the critical plane approach is used to predict the fatigue lives of composite laminates, the laminates are usually