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Reliability analysis based on a novel density estimation method for structures with correlations

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Abstract Estimating the Probability Density Function (PDF) of the performance function is a direct way for structural reliability analysis, and the failure probability can be easily obtained by integration in the failure domain. However, efficiently estimating the PDF is still an urgent problem to be solved. The existing fractional moment based maximum entropy has provided a very advanced method for the PDF estimation, whereas the main shortcoming is that it limits the application of the reliability analysis method only to structures with independent inputs. While in fact, structures with correlated inputs always exist in engineering, thus this paper improves the maximum entropy method, and applies the Unscented Transformation (UT) technique to compute the fractional moments of the performance function for structures with correlations, which is a very efficient moment estimation method for models with any inputs. The proposed method can precisely estimate the probability distributions of performance functions for structures with correlations. Besides, the number of function evaluations of the proposed method in reliability analysis, which is determined by UT, is really small. Several examples are employed to illustrate the accuracy and advantages of the proposed method.

1. Introduction

Reliability analysis is used to assess the safety of an engineering system or structure, and reliability is defined as the probability that a system, subsystem, or device will perform adequately for a specified period of time under specific operating conditions.[1] Commonly, the performance function of a system or structure is characterized by a function $G(\boldsymbol{X})$, which is a function of physical random variables $\boldsymbol{X} = [X_1\ X_2 \ldots X_n]$ (n is the dimensionality of the input vector). The definition equation of the failure probability, P_f, is given as

$$P_\mathrm{f} = \int_F f_{\boldsymbol{X}}(\boldsymbol{x})\mathrm{d}\boldsymbol{x} \tag{1}$$

where $f_{\boldsymbol{X}}(\boldsymbol{x})$ is the joint Probability Density Function (PDF) of the input variables $\boldsymbol{X}$, and $F = \{\boldsymbol{X} : G(\boldsymbol{X}) \leqslant 0\}$ is the failure

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region. During the past several decades, various reliability analysis methods have been proposed and developed with the difficulty to compute the failure probability, which can be classified into three groups generally, i.e., moment based analytical methods, sampling based numerical simulation methods, and surrogate model based methods.

Moment based analytical methods have been firstly developed, and they assess system safety by the reliability index, which is a function of a few integral moments of the performance function. Among them, the First-Order Reliability Method (FORM) and Second-Order Reliability Method (SORM)[2–4] are most classical and widely used. Both the FORM and SORM aim at searching for a design point, which locates on the limit state surface $G(\boldsymbol{X})=0$ and is closest to the origin of coordinate in a standard normal space. Then the original performance function is approximated by a low-order function at the obtained design point. Obviously, the accuracy of these two methods lies heavily on the search of the design point and the approximation of the performance function, and it would spend high costs to obtain the derivative information of the performance function with respect to input variables when searching for the design point using iteration algorithms for complicated engineering problems. Consequently, moment based analytical methods may have large errors when dealing with highly dimensional or implicit problems.

Sampling based numerical simulation methods can artfully avoid the above shortcomings, and they are suitable for most of the reliability problems, with no constraint on the type of performance function. Doubtlessly, the Monte Carlo Simulation (MCS) method is a representative numerical simulation method, and it is easy to implement and probably the most widely used method for reliability analysis. However, it becomes quite inefficient when dealing with those rare events. In order to obtain convergent results, the simulation sampling size should be large enough, generally, $(10^2-10^4)/P_{\mathrm{f}}$ points should be sampled. Especially, for engineering problems with implicit performance functions, large numbers of mode simulations are unpractical and the computational cost is really unaffordable. The Importance Sampling (IS) method[5] is a popular variance reduction technique, and its computational efficiency has been improved compared with MCS. Meanwhile, for engineering problems with implicit performance functions and small failure probabilities, IS is also incompetent and the accuracy relies on the estimated design point. In conclusion, sampling based numerical simulation methods show low efficiency, but it is worth emphasizing that this family of methods is always employed to test the accuracy and efficiency of newly developed approaches.

Considering the computational burden of numerical simulation methods, surrogate model based methods have been widely researched. They aim at utilizing surrogate models to substitute the real performance functions, and thus the computational burden of evaluating the implicit performance functions can be reduced obviously. Many surrogate techniques have been developed up to now, and commonly used surrogate models include the response surface model,[6,7] the artificial neural networks,[8,9] the support vector machine,[10,11] and the Kriging model.[12–14] Generally, the accuracy of results obtained by these surrogate model based methods relies on the accuracy of the surrogate models, and research on the balance between efficiency and accuracy when using this type of methods has attracted increasing attention.

In recent years, a major breakthrough has been achieved in estimating the PDF of the response function of a system or structure using the concept of entropy, a measure of uncertainty. Under some given moment constraints of the response function, the Principle of Maximum Entropy (P-ME) proposed by Jaynes[15] can estimate the PDF of the response function by a way of maximizing the entropy. Meanwhile, in order to model the distribution tail of the PDF of the response function accurately, a larger number of moments are required. Zhang and Pandey pointed out in Ref. 16 that the entropy maximization algorithm shows the numerical instability well as the number of moment constraints increases and the tail of the obtained PDF may become an oscillatory function. Significantly, fractional moments have promoted the development of the P-ME.[16–20] Zhang and Pandey[16] proved that a fractional moment contains information of a large number of integral moments. The concepts of the P-ME, fractional moment, and dimensional reduction method were used to accurately estimate the PDF of the structural response function, and the failure probability with high precision could be easily calculated based on the available PDF. There is no doubt that the method proposed by Zhang and Pandey[16] is really efficient. A Multiplicative Dimensional Reduction Method (M-DRM) based on the concept of high-dimensional model representation was proposed by Zhang and Pandey[16] to transform the original response function into the form of a product of univariate functions, and thus the fractional moments could be easily computed by the integrations of one-dimensional functions using the Gaussian integration scheme. Consequently, only a few functional evaluations are essential for structural reliability analysis. However, some shortcomings and limitations have been detected with the method proposed by Zhang and Pandey[16] through deep research, which come from the M-DRM actually, as shown in Section 2.2. The major limitation is that it can only deal with reliability problems with mutually independent input variables. While in many cases, dependent input variables exist in structural systems,[21–25] thus it is very necessary to extend the fractional moment based P-ME method to the correlated field.

Note that a new technique, called Unscented Transformation (UT), was applied by Julier and Uhlmann[26] to propagate mean and covariance information through nonlinear transformations. A set of weighted sigma points are chosen deterministically, considering the mean and covariance of the sigma points must match those of the prior distribution to be transformed. Researchers have applied UT in many fields, such as Kalman filter,[26] statistical robust design,[27] wind production system,[28] and sensitivity analysis,[29]. Among them, UT was used to compute the mean and covariance of outputs, namely lower integral moments. In this paper, we apply UT to calculate the fractional moments of the performance function, so as to extend the fractional moment based P-ME to the correlated field, and widen its engineering applicability.

The remainder of the paper is organized as follows. Section 2 briefly reviews the fractional moment based P-ME and presents some discussions about this method. Section 3 describes the UT method and gives the computational effort of the proposed method. In Section 4, numerical and engineering examples are introduced and analyzed. Finally, Section 5 draws conclusions.