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Optimal Sensor Placement for Underwater Target Positioning with Noisy Range Measurements

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Abstract: Autonomous underwater vehicles (AUVs) are becoming ubiquitous due in part to the flexibility and versatility that a number of them display in the execution of individual and cooperative tasks. These characteristics, coupled with the fact that their use avoids placing human lives at risk, makes them quite attractive in a number of missions. Central to the operation of some classes of AUVs is the availability of a good underwater positioning system to localize one or more vehicles simultaneously based on information received on-board a support ship or an autonomous surface system. Speaking in loose terms, we are interested in determining the optimal geometric configuration of a surface sensor network that will, in a well defined sense, maximize the range-related information available for underwater target positioning. To this effect, we assume that the range measurements are corrupted by white Gaussian noise, the variance of which is distance-dependent. The Fisher Information Matrix and the maximization of its determinant are used to determine the sensor configuration that yields the most accurate positioning of the target. An explicit analytical result is first obtained for a three sensor network and then extended to a generic 'n' sensor network. It is shown that the optimal configuration lends itself to an interesting geometrical interpretation and that the "spreading" of the configuration depends directly on the intensity of the range measurement noise and the target depth. Simulation examples illustrate the key results derived.

Keywords: Autonomous underwater vehicles, Fisher information matrix, Cramer-Rao Bound, optimization, positioning, sensor networks.

1. INTRODUCTION

The use of autonomous underwater vehicles (AUVs) in different research and commercial areas has been increasing in the last few years. For reasons that have to do with autonomy, flexibility, and the new trend in miniaturization, AUVs are steadily emerging as tools par excellence to replace ROVs and also humans in the execution of many demanding tasks at sea that include pipeline inspection, seabed surveying, and archaeological research, to name but a few. Furthermore, their use in collaborative tasks allows for the realization of complex missions, often with relatively simple systems; see Ghabcheloo (2009) and Yamaguchi (2004).

Central to the operation of some classes of AUVs is the availability of good underwater positioning systems to localize one or more vehicles simultaneously based on information received on-board a support ship or an autonomous surface system. The info thus obtained is at times used to follow the state of progress of a particular mission or, if reliable acoustic modems are available, to relay it as a navigation aid to the navigation systems existent on-board the AUV. Similar comments apply to a (yet to be developed) generation of positioning systems to aid in the tracking of one or more divers. Motivated by similar developments in ground robotics, we address the problem of single target positioning based on measurements of the ranges between the target and a set of sensors at the surface, obtained via acoustic ranging devices. In particular, and speaking in loose terms, we are interested in determining the optimal configuration (formation) of a surface sensor network that will, in a well defined sense, maximize the range-related information available for underwater target positioning. To this effect, we assume the range measurements are corrupted by white Gaussian noise, the variance of which is distance-dependent. The computation of the target position may be done by resorting to trilateration algorithms. See for example Alcocer (2009), Alcocer (2007), Bar-Shalom et al. (2001), and the references therein for an introduction to this circle of ideas, covering both theoretical and practical aspects.

The problem that we address was greatly inspired by the work reported in Martinez & Bullo (2006) on optimal ranging sensor placement to improve the accuracy in the localization of ground robots. Given a localization strategy, the optimal sensor configuration can be ascertained by examining the corresponding Cramer-Rao Bound (CRB) or Fisher Information Matrix (FIM). In this paper, following the methodology described in Martinez & Bullo

(2006), we use the determinant of the FIM as an indicator of the performance that is achievable with a given sensor configuration. Maximazing this quantity yields the most appropriate sensor formation geometry.

Given a target localization problem, the optimal geometry of the sensor configuration depends strongly on the constraints imposed by the task itself (e.g. maximum number and type of sensors that can be used) and the environment (e.g. ambient noise). An inadequate sensor configuration may yield large localization errors. It is interesting to remark that even though the problem of optimal sensor placement for range based localization is of great importance, not many results are available on this topic yet. Exceptions include a series of interesting results that go back to the work of Abel (1990), where the Cramer-Rao bound is used as an indicator of the accuracy of source position estimation and a simple geometric interpretation of that bound is offered. In the same reference, the authors describe a solution to the problem of finding the sensor arrangements that minimize the bound, subject to geometric constraints. In particular, "Carter's optimal arrays yielding minimum range, bearing and position bound variance subject to the constraint that the sensors lie along a line segment are determined without tedious algebraic manipulations". In Levanon (2000), the problem of position determination in two-dimensional (2-D) scenarios is examined. The author shows explicitly what the lowest possible geometric dilution of precision (GDOP) attainable from range or pseudo-range measurements to N optimally located points is and determines the corresponding regular polygon-like sensor configuration. Aranda et al. (2005) study optimal sensor placement and motion coordination strategies for mobile sensor networks. For a target tracking application with range sensors, they investigate the determinant of the FIM and compute it in the 2D and 3D cases. They further characterize the global minimal of the 2D case. In Jourdan & Roy (2008), an iterative algorithm that places a number of sensors so as to minimize the position error bound is developed, yielding configurations for the optimal formation subject to several complex constraints. Finally, Isaacs et al. (2009) address the problem of localizing a source from noisy time-of-arrival measurements by seeking an extreme of the FIM for truncated, radially-symmetric source distributions.

Motivated by previous work, in this paper we address the problem of finding the optimal geometric configuration of a surface-based sensor formation for the localization of an underwater target, based on target-sensor range measurements only. Notice that the sensors are restricted to lie at the sea surface. A problem of this kind was previously studied in Zhang (1995), where a method to determine the optimal two dimensional spatial placement of multiple sensors participating in a robot perception task was introduced. One of the scenarios considered was that of localizing an underwater vehicle, where the locations of the acoustic receivers are constrained to lie on a horizontal plane.

The key contributions of the present paper are twofold: i) a global solution to the optimal sensor configuration problem is obtained analytically, and ii) in striking contrast to what is customary in the literature, where zero mean Gaussian processes with fixed variances are assumed for the range measurements, the variances are now allowed to depend explicitly on the ranges themselves. This allows us to capture the fact that measurement noise increases (albeit in a nonlinear manner) with the distances measured.

The document is organized as follows. Section 2 derives the FIM for the optimal sensor placement problem and computes its determinant for the case where the measurement noise is Gaussian, with distance-dependent variance. The optimal configuration for a three sensor formation is obtained in Section 3, for zero mean Gaussian measurement noise with constant covariance. Section 4 derives the optimal sensor configuration considering that the measurement noise covariance is range (distance) dependent. The results of Section 4 are extended to a 'n' sensor formation in Section 5. Finally, in Section 6 simulation examples are shown. Conclusions and topics for further research are included in Section 7.

2. INFORMATION INEQUALITY WITH DISTANCE-DEPENDENT MEASUREMENT NOISE

Underwater range measurements between two objects are plagued with errors that depend on a multitude of effects: depth-dependent speed of propagation of sound in the water, physical propagation barriers, ambient noise, and degrading signal-to-noise ratio as the distance between the two objects increases, to name but a few. For analytical tractability, it is commonly assumed that measurement errors can be captured by Gaussian, zero mean additive noise with constant covariance. See for example Zhang (1995), where different noise covariances are taken for different sensors, but the covariances are constant. Clearly, this assumption is artificial, in view of the simple fact that the "level of noise" is distance dependent. In this paper, in an attempt to better capture physical reality, we assume that the measurement noise can be modelled by a zero-mean Gaussian process with an added term that depends on the distance between the two objects that exchange range data. A similar error model is considered in Jourdan & Roy (2008) for its iterative algorithm. Stated mathematically,

$$w = (I + \eta\delta(r)) \cdot w_0 \tag{1}$$

where ω is measurement noise, ω_0 is a zero mean Gaussian process $N(0, \Sigma_0)$ with $\Sigma_0 = \sigma^2 \cdot I$, I is the identity matrix, r is range, and η is the modelling parameter for the distance-dependent noise component. In the above, δ is the operator *diag*, that either converts a square matrix into a vector consisting of its diagonal elements, or converts a vector into a square diagonal matrix whose diagonal components are the array elements. With these assumptions, the measurement noise covariance is given by

$$\Sigma = \sigma^2 \left(I + \eta\delta(r)\right)^2 \tag{2}$$

Furthemore, letting q be the target position, p_i the position of the ith acoustic ranging sensor, and w_i the corresponding measurement noise, the measurement model is

$$z_i(q) = h_i(\|(p_i - q)\|) + w_i \tag{3}$$

where $h_i(\|(p_i - q)\|) = \|(p_i - q)\| = r_i$.