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A method of determining effective elastic properties of honeycomb cores based on equal strain energy

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Abstract A computational homogenization technique (CHT) based on the finite element method (FEM) is discussed to predict the effective elastic properties of honeycomb structures. The need of periodic boundary conditions (BCs) is revealed through the analysis for in-plane and out-of-plane shear moduli of models with different cell numbers. After applying periodic BCs on the representative volume element (RVE), comparison between the volume-average stress method and the boundary stress method is performed, and a new method based on the equality of strain energy to obtain all non-zero components of the stiffness tensor is proposed. Results of finite element (FE) analysis show that the volume-average stress and the boundary stress keep a consistency over different cell geometries and forms. The strain energy method obtains values that differ from those of the volume-average method for non-diagonal terms in the stiffness matrix. Analysis has been done on numerical results for thin-wall honeycombs and different geometries of angles between oblique and vertical walls. The inaccuracy of the volume-average method in terms of the strain energy is shown by numerical benchmarks.

1. Introduction

For the past several decades, sandwich plates with a honeycomb core have been widely used in the field of aviation. In understanding the behavior of sandwich structures under different types of load, the honeycomb core is often regarded as homogeneous solid with orthotropic elastic properties.[1] As a result, research on the effective elastic properties of the honeycomb core is of great essence for the calculation and design of honeycomb sandwich structures.

A computational homogenization technique (CHT) has been found to be a powerful method to predict the effective properties of structures with periodic media. In order to obtain the effective stiffness tensor, which relates to the equivalent strain and stress, this process is divided into solving six elementary boundary value problems, which refer to uniaxial tensile and shear in three directions.[2–5] The equivalent strain is determined after applying the unit displacement boundary conditions (BCs) on the representative volume element (RVE) cell corresponding to one of the six elementary problems. Different

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methods have been used when dealing with the equivalent stress. Volume-average stress is often used to determine effective properties in the literature.[2–6] Catapano and Montemurro[2] investigated the elastic behavior of a honeycomb with double-thickness vertical walls over a wide range of relative densities and cell geometries. The strain-energy based numerical homogenization technique was also used by Catapano and Jumel[3] in determining the elastic properties of particulate-polymer composites. Montemurro et al.[4] performed an optimization procedure at both meso and macro scales to obtain a true global optimum configuration of sandwich panels by using the NURBS curves to describe the shape of the unit cell. Malek and Gibson[5] got numerical results of a thick-wall honeycomb closer to their analytical solutions by considering nodes at the intersections of vertical and inclined walls. Shi and Tong[6] focused on the transverse shear stiffness of honeycomb cores by the two-scale method of homogenization for periodic media. Many researchers also seek the stress on the boundary of the RVE cell. Li et al.[7] used the sum of the node force on the boundary of the RVE cell to obtain the equivalent stress. Papka and Kyriakides[8] set plates on the top and bottom of the RVE cell to exert BCs. However, regarding honeycomb structures as a combination of cell walls and air, the stress variations on the boundary cause the boundary stress inaccurate to calculate effective properties. Some divergence still exists in numerical results of regular hexagonal honeycomb structures with analytical solutions, especially for the in–plane and out-of-plane shear moduli. From the definitions of effective elastic properties expressed by Yu and Tang,[9] the equivalent stress is required to make sure that the RVE cell and the corresponding unit volume of the homogeneous solid undergo the same strain energy. Hence, the whole honeycomb structure containing a finite number of RVE cells have the same strain energy as that of the whole volume of the homogeneous solid. The mathematical homogenization theory (MHT) has proven that the strain energy in the RVE can be determined by the volume-average stress and strain.[10] However, it is not always suitable for the calculation of the volume-average stress method in the CHT. The volume-average method cannot get all precise values in the stiffness matrix, and it is found to get larger strain energy than that obtained from direct analysis in two-dimensional porous composites by Hollister and Kikuchi.[11] Therefore, we focus on the total strain energy of the RVE cell and propose a new method to determine all the components of the stiffness tensor more accurately in terms of the strain energy.

In Section 2, the differences between the proposed energy method and previous methods are analyzed. A process to obtain 9 components of the effective stiffness tensor based on the energy method is introduced. Then, finite element (FE) models are discussed in Section 3. Convergence analysis has been done over material properties, mesh sizes, and BCs applied on the whole model. In addition, two models are proposed to acquire in–plane and out-of-plane shear moduli according to the different deformations of a single RVE cell and a finite number of RVE cells under the same loading. After establishing appropriate models for honeycomb structures, numerical results over a range of cell geometries are compared to analytical solutions in literature in the next section. Finally, Section 5 ends the paper with some conclusions.

2. Prediction method

2.1. Introduction of a computational homogeneous technique

Previous experimental data and theory have proven that a honeycomb core can be classified as an orthotropic material.[12] Under this assumption, a honeycomb core conforms to generalized Hook's law[13] as

$$\begin{bmatrix} \bar{\sigma}_{11} \\ \bar{\sigma}_{22} \\ \bar{\sigma}_{33} \\ \bar{\sigma}_{23} \\ \bar{\sigma}_{13} \\ \bar{\sigma}_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \bar{\varepsilon}_{11} \\ \bar{\varepsilon}_{22} \\ \bar{\varepsilon}_{33} \\ \bar{\gamma}_{23} \\ \bar{\gamma}_{13} \\ \bar{\gamma}_{12} \end{bmatrix} \tag{1}$$

where $\bar{\sigma}$ and $\bar{\varepsilon}$ are, respectively, the equivalent stress and strain tensors for the whole geometry of an RVE cell. C_{ij} is one of the components of the stiffness tensor $\boldsymbol{C}$ which is symmetric as $C_{ij} = C_{ji}$. In addition, the shear strain relates the components of the strain tensor as follows,

$$\begin{cases} \bar{\varepsilon}_{11} \rightarrow \bar{\varepsilon}_{11} = \frac{\partial u}{\partial x} \\ \bar{\varepsilon}_{22} \rightarrow \bar{\varepsilon}_{22} = \frac{\partial v}{\partial y} \\ \bar{\varepsilon}_{33} \rightarrow \bar{\varepsilon}_{33} = \frac{\partial w}{\partial z} \\ \bar{\gamma}_{23} \rightarrow 2\bar{\varepsilon}_{23} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \\ \bar{\gamma}_{13} \rightarrow 2\bar{\varepsilon}_{13} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \\ \bar{\gamma}_{12} \rightarrow 2\bar{\varepsilon}_{12} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \end{cases} \tag{2}$$

where u, v, and w represent the displacements in the x, y, and z directions.

To determine the effective stiffness matrix of the RVE cell, six elementary BCs are applied on the RVE cell, which refer to three uniaxial extensions and three shear deformations. For each load case, only one component of the strain tensor is not zero. Then the relative stiffness component is determined by the equivalent stress. Take C_{11} for example,

$$C_{11} = \frac{\bar{\sigma}_{11}}{\bar{\varepsilon}_{11}} \quad \text{in load case} \quad \bar{\varepsilon}_{11} \neq 0 \tag{3}$$

After obtaining all the 9 independent components in the stiffness matrix, engineering constants can be derived from the compliance matrix which is the inverse of the stiffness matrix.

2.2. Energy method

Assuming that an elementary shear boundary displacement is applied on the RVE cell ($\bar{\gamma}_{kl} \neq 0$), Eq. (4) is tenable since the boundary of the RVE cell has an identical displacement.

$$\frac{1}{V} \int \gamma_{kl} \mathrm{d}v = \bar{\gamma}_{kl} \tag{4}$$

where V represents the total volume of the RVE and subscript "kl" stands for the certain BC.

The strain energy of the RVE cell under certain loading can be determined by the FE result as

$$U = \frac{1}{2} \int \sigma_{ij} \varepsilon_{ij} \mathrm{d}v \quad i, j = 1, 2, 3 \tag{5}$$