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Adaptive formation control of quadrotor unmanned aerial vehicles with bounded control thrust

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Abstract In this paper, the flight formation control problem of a group of quadrotor unmanned aerial vehicles (UAVs) with parametric uncertainties and external disturbances is studied. Unit-quaternions are used to represent the attitudes of the quadrotor UAVs. Separating the model into a translational subsystem and a rotational subsystem, an intermediary control input is introduced to track a desired velocity and extract desired orientations. Then considering the internal parametric uncertainties and external disturbances of the quadrotor UAVs, the priori-bounded intermediary adaptive control input is designed for velocity tracking and formation keeping, by which the bounded control thrust and the desired orientation can be extracted. Thereafter, an adaptive control torque input is designed for the rotational subsystem to track the desired orientation. With the proposed control scheme, the desired velocity is tracked and a desired formation shape is built up. Global stability of the closed-loop system is proven via Lyapunov-based stability analysis. Numerical simulation results are presented to illustrate the effectiveness of the proposed control scheme.

1. Introduction

In recent years, formation control problems have become one of the leading research areas among the research community. Compared with a single vehicle, a group of vehicles that work cooperatively may provide a better performance and fulfill a more difficult task, which has been motivating further studies of formation control problems. In order to achieve formation control, consensus algorithms for multi-agent systems have been extensively studied in the literature,[1–11] and can be categorized as leader-follower,[1–4] behavior-based,[5–7] and virtual structure.[8–11]

Nevertheless, these works of consensus algorithms referred above mainly considered simple models such as single or double integrators. Since a quadrotor unmanned aerial vehicle (UAV) is a more complex system with characteristics of multi-variable, nonlinear, strong coupling, and under-actuation, its formation control problems are more complicated than those of simple models. Fortunately, with the devel-

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opment of technology and the increasing demand for formation control of robot systems, researchers are trying to deal with formation problems for complex systems using these approaches.

A consensus algorithm is designed[3] to enable quadrotor UAVs to fly in formation in a leader-follower structure, where artificial potential field functions are used to ensure collision avoidance. Galzi and Shtessel[7] proposed a sliding mode controller that achieved UAV formation tracking affected by unknown bounded disturbances, and three control strategies were presented: the "Tracking" strategy, the "Placement" strategy, and the "Anti-Collision" strategy.

Guerrero et al.[9] studied the quadrotor UAV formation problem with input saturation. Assuming that the angles are operated in a neighborhood of an origin, the model of a quadrotor UAV is simplified as a fourth-order integrator. A bounded control law was designed using the nested saturation technique. In addition, formation control problems with network communication delays have been studied, to control a team of micro-aerial vehicles moving quickly with time delays and communication failures in the network[8], and to deal with bounded time-varying delays that can be arbitrarily fast using frequency domain design methods.[10] Wu and Shi[11] studied the consensus in multi-agent systems with random delays governed by a Markov chain, and found that with a sufficient condition given by a set of linear matrix inequalities (LMIs), the delays can be tolerated.

In practical situations, the mass of a quadrotor UAV is not fixed when equipped with different sensors for different missions or due to a payload configuration change. Hence, intrinsic parameter uncertainties should be taken into consideration. Unfortunately, model parameters are generally uncertain and cannot be measured precisely. The general backstepping control method that depends on accurate system model parameters is invalid when facing model uncertainties. Besides, it will be unrealistic to measure the mass value and the moment of inertia value of the vehicle during a flight. Therefore, some adaptive controllers[12–17] for controlling a single vehicle are proposed to deal with uncertainties.

Apparently, extending the existing strategies proposed for simple multi-agent systems and single quadrotor UAVs to the formation control problem of quadrotor UAVs is not a trivial task. Although the control problems of a single quadrotor UAV with parameter uncertainties have been well discussed, the quadrotor UAV formation problem with uncertainties is not well taken into consideration.

The main contribution of this paper is to provide a formation control scheme for a group of quadrotor UAVs with parametric uncertainties and external disturbances. To deal with under-actuation of the quadrotor UAVs in the formation, a prior-bounded intermediary control input and the adaptive laws of the mass uncertainty and the external force disturbances are designed for position tracking, by which the bounded control thrust and the bounded desired orientation are extracted. Then, a control torque and the adaptive laws of the moment of inertia uncertainties and the external torque disturbances are designed to track the desired orientation in terms of the unit-quaternions. A novel Lyapunov function is constructed to determine the stability of the overall system.

The remainder of this paper is organized as follows. In Section 2, the dynamic model of the quadrotor UAVs and some definitions and assumptions are given. In Section 3, the control laws and adaptive laws are proposed. In Section 4, global stability of the closed-loop system is proven via Lyapunov-based analysis. In tion 5, simulation results are given to demonstrate the effectiveness of the control laws proposed above. Finally, the conclusions are given in Section 6.

2. System model and preliminaries

2.1. System model

Let the subscript "j" ($j = 1, 2, \ldots, n$) denote the jth quadrotor UAV in the formation, where n is the total number of the quadrotor UAVs; $E = \{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$ denote the inertial frame, with $\boldsymbol{e}_1 = [1, 0, 0]^{\mathrm{T}}$, $\boldsymbol{e}_2 = [0, 1, 0]^{\mathrm{T}}$ and $\boldsymbol{e}_3 = [0, 0, 1]^{\mathrm{T}}$ the unit vectors in the directions x, y, z of the frame E; $B_j = \{\boldsymbol{b}_{j1}, \boldsymbol{b}_{j2}, \boldsymbol{b}_{j3}\}$ denote the body-fixed frame of the jth quadrotor UAV, with $\boldsymbol{b}_{j1} = [1, 0, 0]^{\mathrm{T}}$, $\boldsymbol{b}_{j2} = [0, 1, 0]^{\mathrm{T}}$ and $\boldsymbol{b}_{j3} = [0, 0, 1]^{\mathrm{T}}$ the unit vectors in the directions x, y, z of the frame B_j; $\boldsymbol{p}_j \in \mathbf{R}^3$ and $\boldsymbol{v}_j \in \mathbf{R}^3$ are the position and linear velocity expressed in the inertial frame; $\boldsymbol{\omega}_j \in \mathbf{R}^3$ is the angular velocity; the orientation is represented by unit-quaternions defined by $\boldsymbol{Q}_j = [\boldsymbol{q}_j^{\mathrm{T}}, \eta_j]^{\mathrm{T}}$, composed of a vector component $\boldsymbol{q}_j = [q_{j1}, q_{j2}, q_{j3}]^{\mathrm{T}} \in \mathbf{R}^3$ and a scalar component η_j, which are constrained by

$$\boldsymbol{q}_j^{\mathrm{T}} \boldsymbol{q}_j + \eta_j^2 = 1 \tag{1}$$

The multiplication between two unit quaternions, $\boldsymbol{Q}_1 = [\boldsymbol{q}_1^{\mathrm{T}}, \eta_1]^{\mathrm{T}}$ and $\boldsymbol{Q}_2 = [\boldsymbol{q}_2^{\mathrm{T}}, \eta_2]^{\mathrm{T}}$, is defined by

$$\mathbf{Q}_1 \odot \mathbf{Q}_2 = \begin{bmatrix} \eta_1 \boldsymbol{q}_2 + \eta_2 \boldsymbol{q}_1 + \boldsymbol{S}(\boldsymbol{q}_1)\boldsymbol{q}_2 \\ \eta_1 \eta_2 - \boldsymbol{q}_2^{\mathrm{T}} \boldsymbol{q}_1 \end{bmatrix} \tag{2}$$

where $\odot$ is a non-commutative multiplication operator, and $\boldsymbol{S}(\bullet)$ is the skew-symmetric matrix operator defined by

$$\boldsymbol{S}(\boldsymbol{a}) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{3}$$

for a vector $\boldsymbol{a} = [a_1, a_2, a_3]^{\mathrm{T}}$.

The physical structure of the quadrotor UAV is illustrated in Fig. 1.

For convenience, we divide the system into two subsystems: Σ_j^1 is the translational subsystem and Σ_j^2 is the rotational

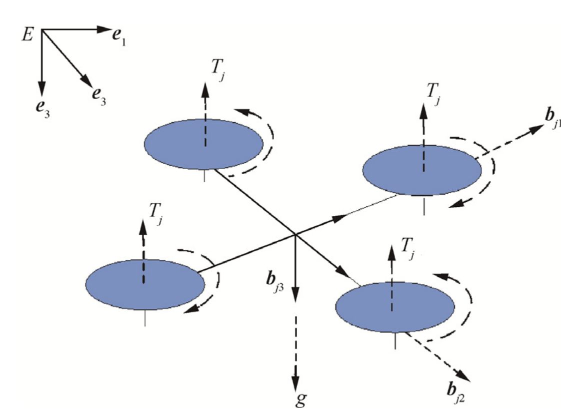

Fig. 1 Physical structure of a quadrotor UAV.

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