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Variable stiffness design of redundantly actuated planar rotational parallel mechanisms

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Abstract Redundantly actuated planar rotational parallel mechanisms (RAPRPMs) adapt to the requirements of robots under different working conditions by changing the antagonistic internal force to tune their stiffness. The geometrical parameters of the mechanism impact the performances of modulating stiffness. Analytical expressions relating stiffness and geometrical parameters of the mechanism were formulated to obtain the necessary conditions of variable stiffness. A novel method of variable stiffness design was presented to optimize the geometrical parameters of the mechanism. The stiffness variation with the internal force was maximized. The dynamic change of stiffness with the dynamic location of the mechanism was minimized, and the robustness of stiffness during the motion of the mechanism was ensured. This new approach to variable stiffness design can enable off-line planning of the internal force to avoid the difficulties of on-line control of the internal force.

1. Introduction

Planar parallel manipulators perform two translations along the *x*- and *y*-axes, and rotate through an angle around the *z*-axis, perpendicular to the plane. They have some potential advantages over serial robotic manipulators such as better accuracy, greater load capacity, and higher velocity and acceleration.[1,2] The redundantly actuated planar rotational parallel mechanism (RAPRPM) is a special type of planar parallel manipulator. It does not have the ability to move along the *x*- and *y*-axes, and only has a single degree of freedom, rotating around the *z*-axis. Meanwhile, the stiffness of rotation around the *z*-axis can be modulated by employing redundant actuation. The performances including inverse kinematics, forward kinematics, Jacobian matrix, workspace, singularity, and dexterity of planar parallel manipulators have been analyzed.[1–4] Stiffness modeling of a robotic manipulator is also one of the important issues that allows a user to evaluate its compatibility for certain tasks.[5] Based on biological studies of the muscular properties and the skeletal structures of fish, Cui and Jiang presented a robotic fish consisting of planar serial-parallel mechanisms, i.e., the RAPRPMs connecting to each other in series. It included rigid bodies, springs, dampers, and revolution joints. Their results showed that the swimming

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performance of the robotic fish was largely dependent on the body stiffness and the driven frequency.[6] Biological experiments of fish have shown that fish change their natural frequency by modulating the stiffness of their bodies to match the driving frequency. Then fish can employ resonance to improve their swimming efficiency.[7–9] Swimming fish that can tune their body stiffness by appropriately timed muscle contractions are able to maximize peak acceleration or swimming speed. The muscles are modeled as springs of constant stiffness.[10] To study the body stiffness of robotic fish consisting of planar serial-parallel mechanisms, it is important to study the stiffness design and stiffness control of the RAPRPM. Stiffness control schemes realized by employing redundant actuation can be broadly categorized as: passive stiffness control (PSC), feedback stiffness control (FSC), and active stiffness control (ASC).[11,12] PSC is a scheme that changes the stiffness of the mechanism by adding flexible elements to the original mechanism.[13,14] Because the stiffness of flexible elements cannot be changed much, the stiffness of the mechanism is changed less by using PSC. An FSC scheme chooses proportional coefficients in the positioning joint controllers that correspond to the desired characteristics for control of the end-effector.[15,16] However, the modulation of proportional coefficients of controllers may make the system unstable. An ASC scheme yields antagonistic forces in a redundantly actuated mechanism. The internal forces balance each other in a closed mechanism and do not perform any effective work, but generate end-effector stiffness.[17–20] An ASC scheme can significantly modulate the stiffness by modulating the internal force, which involves off-line planning of antagonistic actuator loads, so that one can obtain the desired object stiffness.[21–24] An ASC scheme is chosen to control the stiffness of the RAPRPM because of its advantages over other stiffness control schemes.

In addition, it is also meaningful to apply an ASC scheme to maintain constant stiffness and maximize the change of stiffness with the internal force. Sungcheul et al. introduced two indexes, one of which was suggested to make the minimum stiffness similar to the maximum stiffness at a given point and to ensure robustness and balance of the stiffness in all directions. The other index was used to maximize the stiffness in a fixed direction along the pathway.[25] Hence, for the RAPRPM whose stiffness changes with the dynamic location of a platform, applying an ASC scheme to enable on-line control of the internal force according to the dynamic location to keep the stiffness constant and to maximize the change of stiffness with the internal force, would increase the controlling difficulty and responding time. However, when applying an ASC scheme to enable off-line planning of the internal force to avoid the difficulties of on-line control, geometrical parameters are required to meet the following three requirements. Firstly, the amount of active stiffness variation with the internal force is maximum. Secondly, the proportion of active stiffness in total stiffness is maximum. Thirdly, the dynamic change of active stiffness with the rotating angle is minimum to ensure the robustness of stiffness during movement of the platform. In addition, optimization strategies such as particle swarm optimization and genetic algorithms have been widely used to minimize the power requirement for a planar parallel manipulator,[26] to compensate for compliance errors,[5] to obtain superior dexterous workspace,[27,28] or to maximize stiffness.[29,30] Similarly, the geometrical parameters are optimized to maximize the stiffness variation with the internal force and minimize the dynamic changes of total stiffness with the dynamic location of the mechanism.

2. Torsional stiffness of RAPRPM

2.1. Variable stiffness principle of RAPRPM

Cui and Jiang presented the structure of a compliant fish consisting of planar serial-parallel redundantly actuated mechanisms, as shown in Fig. 1,[6] in which the RAPRPMs connect each other in series. The capacity of the fish to modulate stiffness can be replicated by changing the stiffness of the RAPRPMs.[6] As one of a series, the working principle of a single RAPRPM is shown in Fig. 2. The top platform A_1OA_2 is supported by the middle rigid leg OB_3 and the elastic legs A_1B_1 and A_2B_2. $|l_1|$ is the length of the elastic leg A_1B_1. $|l_2|$ is the length of the elastic leg A_2B_2. The elastic legs A_1B_1 and A_2B_2 on both sides connect the rotating pairs of the upper revolute joints A_1 and A_2 on the top platform and the rotating pairs of the lower revolute joints B_1 and B_2 on the fixed platform $B_1B_2B_3$. r_a is the center distance of the upper revolute joints, while r_b is the center distance of the lower revolute joints. The middle rigid leg OB_3 is attached to the fixed platform. The top platform rotates around the rotating center O with a single degree-of-freedom. q is the rotating angle, the position where the rotating angle $q = 0$ rad is defined as the initial position, h is the distance between the rotating center O and the fixed platform, L_c is the distance between the rotating center O and the top platform, and $\boldsymbol{r}$ is the vector from the rotating center O to the upper revolute joints. The linear drivers C_1 and C_2 change the internal forces resulting from the elastic legs A_1B_1 and A_2B_2, respectively, and the internal forces balance each other to provide active stiffness in the closed mechanism. $\boldsymbol{f}$ is the outputting internal force of the leg. XOY is defined as the base coordinate fixed to the rotating center O, and its Y-axis parallels to OB_3. x_0Oy_0 is defined as the rotating coordinate fixed to the top platform.

The torsional stiffness K of the RAPRPM is defined as[25]

$$K = \frac{\partial Q}{\partial q} \tag{1}$$

where Q is the torque of the top platform.

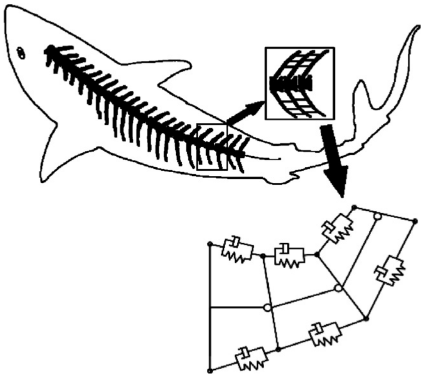

Fig. 1 Compliant fish with serial-parallel redundantly actuated mechanisms.

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