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Enhanced wave and finite element method for wave propagation and forced response prediction in periodic piezoelectric structures

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Abstract As a promising numerical tool of structural dynamics in mid- and high frequencies, the wave and finite element method (WFEM) is receiving increasingly attention and applications. In this paper, an enhanced WFEM has been developed with a reduced model and a new eigenvalue scheme. The reduced model is applicable for structures with piezoelectric shunts or local dampers; the new eigenvalue scheme can mitigate the ill-conditioning when the wave basis is calculated. The enhanced WFEM is applied to a thin-wall structure with periodically distributed piezoelectric materials (PZT). Both free wave characteristics and forced response are analyzed and the influences of the suggested enhancements are presented. It is shown that if the control factors are properly chosen, these enhancements can improve the accuracy while accelerating the calculation. Resulting from the complexity of the application, these enhancements are not optional but imperative.

1. Introduction

For a problem concerning structural dynamics, it is always possible, at least in principle, to arrive at the same conclusions by either mode or wave approach. This equivalence is termed "wave-mode duality" in the literature,[1] and it can be theoretically demonstrated in some simple cases.[1,2] In spite of that, each approach has its own advantages and each provides different views for understanding the same dynamic structural system.

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At low frequencies, a structure can be regarded as a closed system. The structural motion is dominated by well-separated global stationary modes, so it is reasonable to understand the dynamic deformation of a structure as the superposition of modal motions. For a discrete structural model, the number of modes equals the overall number of degrees-of-freedom (DOFs). However, most of the high-order modes hardly contribute to the deformation. It is therefore possible to reduce the size of the problem by truncating modes.[3–5] In terms of vibration control, the guidelines given by the mode approach is to control the vibration of a single mode or multiple modes.[6,7]

Alternatively, in the wave perception, structural deformation is regarded as the superposition of the wave motions, while natural modes are understood as standing waves induced by the reflection of waves on the boundaries.[8] At higher frequencies, the waves could be transmitted through the boundaries and power is radiated or absorbed in the remote parts of the structure.[9] Then the structure is more suitable to be treated as an open system where resonance behavior is less apparent. It is then more reasonable to employ the wave approach. The guideline given by the wave approach for vibration reduction is to modify the wave properties in the interested frequency band so as to dissipate or localize the injected energy.[10,11] This idea has drawn considerable research attention these years with the development of the periodic structures or phononic materials.[12] Since the waves are independent of the boundary conditions, the vibration reduction performance induced by waves is insensitive to boundary conditions as well.[13]

The forced response of a structure can be predicted by the wave approach if the structure is periodic.[14,15] If the structure is partly periodic, for example a structure with several different periodic substructures or a structure with both periodic parts and non-periodic parts, wave approach can still be applied by the diffusion matrix method[16] or hybrid WFEM/FEM (wave and finite element method/finite element method).[17,18] If the structure is near periodic, for example a periodic structure with spatially homogeneous random properties, the wave approaches can also be used. A promising method named stochastic wave finite element[19,20] (SWFE) addresses this problem by a hybridization of the deterministic wave finite element and a parametric probabilistic approach. If the structure is generally non-periodic, a homogenization approach can be performed, leading to an equivalent uniform/periodic structure and the wave approaches can be applied. In this case, the obtained wave characteristics represent the low frequency global behavior of the original structure. The main advantage of using the wave approach is that a periodic structure/substructure can be analyzed by only modeling the smallest repetitive unit cell. In comparison with the full model of the structure/substructure, the dimension of problem is significantly reduced and the computing is therefore accelerated. This feature makes the wave approach a promising method in mid- and high frequency structural analysis[21] when full FE analysis is rather time-consuming.

To manually obtain the desired wave dispersion characteristics, periodically distributed piezoelectric patches with electric circuits have already been considered in the literature.[22–24] Piezoelectric materials have the ability to transform mechanical energy to electrical one and vice versa.[25] This allows one to drastically modify the modal and wave characteristics especially when semi-active circuits are employed.[13,26]

In all these applications concerning wave approach, the core information is the wave characteristics of the structure. That is, at a given frequency, which waves exist in the structure and how they travel (wavenumbers and wave shapes). Analytical formulas can be found for relatively simple cases.[27–29] For periodic structures with complex configurations, for example the uniform thin-wall structure studied by Houillon et al.[30], the analytical solutions are of limited value, particularly at higher frequencies. In recent years, WFEM has been developed to access the wave characteristics of the periodic structures.[15] In WFEM, a unit cell is firstly modeled by FEM and the Bloch theory is then imposed. It finally leads to an eigenvalue problem, yielding the frequency, wavenumbers and wave shapes. WFEM has been applied to 1D periodic structures,[31] plates,[32,33] thin-wall structures,[30] piezoelectric structures[17] and fluid-filled pipes.[34] The experimental studies have also been conducted concerning the wave characteristics of the perforated plates[35], ribbed panels[36] and 1D piezoelectric waveguides.[37] The dispersion curves can be recognized by a spatial Fourier transform of the steady-state response of the finite structure; the results match very well with the numerical results in the aforementioned studies.

However, WFEM still has a series of numerical issues[38,39] including matrix ill-conditioning in free wave analysis and the incorrect estimation of strongly evanescent waves in forced response analysis. Moreover, if the number of DOFs in the FE mesh of the unit cell is numerous, the computing will become slow.

To overcome the matrix ill-conditioning while analyzing the free waves, several eigenvalue schemes have been suggested[10,40] and they mitigate the problem to some extent. To improve the accuracy of the forced response analysis, it is suggested to truncate the wave basis[41,42] so that only the propagating and less-decaying waves are retained. However, the methods have only been validated on rather simple structures. To accelerate the WFEM, reduced models have been developed, and the coordinates transfer matrix can be formed by the wave shapes at selected frequencies[43] or the modal shapes of a unit cell[31] with all the DOFs connecting the adjacent cells fixed. However, the former strongly depends on the selection of waves and the latter is not applicable when there are local dampers or piezoelectric shunts inside the unit cell.

In this paper, an enhanced WFEM has been developed with a reduced model which works for piezoelectric structures. A new eigenvalue formula is proposed to further improve the accuracy of the scheme used in the literature.[10,31,43] The method is applied to a thin-wall structure with periodically distributed piezoelectric patches shunted by identical circuits. The method is validated not only by comparing the dispersion curves but also by checking an energy criterion featured by piezoelectric systems. The forced response of the structure is conducted by WFEM, where the influences of the factors, such as the number of the retained modes, eigenvalue scheme and the number of the kept waves, are separately presented and discussed. Eventually the guidelines for choosing the factors are given.

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