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Study on segmented distribution for reliability evaluation

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Abstract In practice, the failure rate of most equipment exhibits different tendencies at different stages and even its failure rate curve behaves a multimodal trace during its life cycle. As a result, traditionally evaluating the reliability of equipment with a single model may lead to severer errors. However, if lifetime is divided into several different intervals according to the characteristics of its failure rate, piecewise fitting can more accurately approximate the failure rate of equipment. Therefore, in this paper, failure rate is regarded as a piecewise function, and two kinds of segmented distribution are put forward to evaluate reliability. In order to estimate parameters in the segmented reliability function, Bayesian estimation and maximum likelihood estimation (MLE) of the segmented distribution are discussed in this paper. Since traditional information criterion is not suitable for the segmented distribution, an improved information criterion is proposed to test and evaluate the segmented reliability model in this paper. After a great deal of testing and verification, the segmented reliability model and its estimation methods presented in this paper are proven more efficient and accurate than the traditional non-segmented single model, especially when the change of the failure rate is time-phased or multimodal. The significant performance of the segmented reliability model in evaluating reliability of proximity sensors of leading-edge flap in civil aircraft indicates that the segmented distribution and its estimation method in this paper could be useful and accurate.

1. Introduction

As one of the essential devices affecting aircraft safety, the proximity sensors in leading-edge flap is an important part of aircraft operating system. However, proximity sensors in leading-edge flap are also one of the devices with higher failure rate in aircraft.[1] In order to ensure aircraft safety and reduce its maintenance costs, it's necessary to accurately evaluate the reliability of proximity sensors in leading-edge flap.[2]

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However, like most other equipment, the failure rate of the proximity sensors in leading-edge flap is complicated, exhibiting different changes in different operation phases.[3] In order to accurately evaluate the reliability of devices such as proximity sensors in leading-edge flap, whose failure rate is multimodal or multi-stage, only segmented distribution is applicable and inevitable.[4] In this paper, two segmented reliability models are proposed: (1) failure rate is a piecewise linear function about lifetime, namely, segmented linear failure rate mode and (2) failure rate in any stage is of the same formula as the failure rate function of Weibull distribution, called segmented Weibull distribution.

Currently, the application of segmented distribution to reliability evaluation is mainly classified into three categories: describing the degradation process of devices; creating reliability model; and testing and detecting change-points.[5] (1) Now, how to fit the equipment degradation process by segmented distribution is discussed in a lot of literature. For example, Bae et al. [6] proposed a hierarchical Bayesian change-point regression model to fit two-phase degradation patterns and derived the failure-time distribution of a unit randomly selected from its population. Kvam [7] introduced a log-linear model with random coefficients and a change-point to describe the nonlinear degradation path. (2) Since the failure rates of many products may perform different trends at different stages, an application of segmented distribution to evaluating product reliability has been widely concerned. Li et al. [8] estimated the change-point for a piecewise hazard regression model in the presence of right censoring and long-term survivors. In a study by He et al. [9], a sequential testing approach to detect multiple change-points in the hazard function by likelihood ratio statistics and resampling was proposed, which was applicable to both right-censored and interval-censored data. Uhm et al. [10] studied a weighted least squares estimator for Aalen's additive risk model with right-censored survival data which may allow for very flexible handling of covariates. (3) In addition, another important application of segmented distribution is to test whether there are change-points in product failure rate or reliability. For a segmented regression system with an unknown change-point over two domains of a predictor, a new empirical likelihood ratio statistic was proposed to test the null hypothesis of no change by Liu and Qian [11]. Goodman et al. [12] expanded the set of alternatives to allow for multiple change-points, and proposed a model selection algorithm using sequential testing for the piecewise constant hazard model. Suresh [13] derived a test statistic and its asymptotic distribution, and compared the power of the test with other existing tests such as likelihood ratio, Weibull, and log Gamma tests. Nosek and Szkutnik [14] discussed a regression model with a possible structural change and a small number of measurements.

Recently, segmented distribution and its estimation method have caught more attentions. To minimize the expected sum of manufacturing cost, bum-in cost, and warranty cost of failed items found during their warranty period, a cost model was formulated to find the optimal bum-in time based on Weibull hyperexponential distribution by Chou and Tang [15]. Considering different prior densities for parameters and censored survival data, Achcar and Loibel [16] discussed Bayesian analysis on segmented constant hazard function models and put forward their inference methods by Metropolis algorithms. Weibull-exponential distribution, a common segmented distribution, was discussed and its accurate calculation formula by Bayes estimation was proposed by Boukai [17]. Patra and Dey [18] proposed a general class of change-point hazard models for survival data, which included and extended many different types of segmented distribution.

However, the existent segmented distribution is generally considered as a two-segment distribution, or their failure rate is supposed as a constant in recent literature [6–18]. Compared with many other current segmented distributions, the contributions of this paper are as follows. (1) In this paper, not only two kinds of universal n-segment distribution have been discussed, but also two general methods to estimate parameters in the segmented distributions, namely Bayesian estimation and maximum likelihood estimation (MLE), are given. (2) An information criterion adequate for the n-segment distribution is proposed. It can not only test whether a change-point exists or not, but also measure the appropriateness and correctness of the segmented reliability model. (3) A new bathtub curve model is put forward to verify the effects of the segmented distributions.

2. Segmented distribution

In fact, the failure rate of equipment not only is changeable but also often manifests different trends at different phases. Reliability-centered maintenance (RCM) recommended six failure models as shown in Fig. 1.[19] As Fig. 1 shows, apart from failure models C and E, the failure rates of the other failure models show different patterns at different phases, and change-points obviously exist in failure rate curves. Therefore, if one and same distribution is utilized to evaluate a failure whose failure rate is characterized by multiple change-points and multi-peak, evaluation may be rather difficult and inaccurate.[20]

Due to unevenness and inconsistency of failure rate, it is not appropriate to describe the change of reliability by one and same analytical function in the entire lifetime cycle. However, if we divide lifetime into some time intervals according to failure rate trend, the failure rate in every different interval can be described by a different function. Just as piecewise interpolation with a simple function can improve approximation precision, we can more accurately approximate the true change of failure rate. For example, as for failure model A, i.e., bathtub curve, if three different failure rate functions are in correspondence to each phase, then the change of failure rate in the

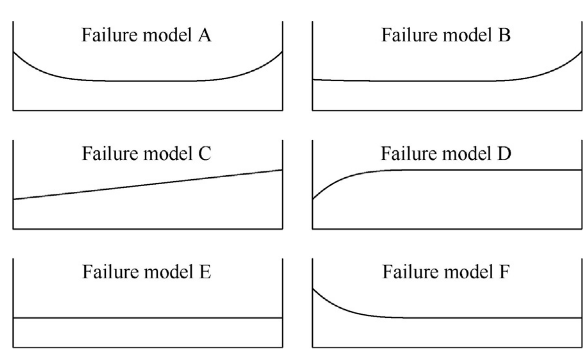

Fig. 1 Six kinds of failure model.