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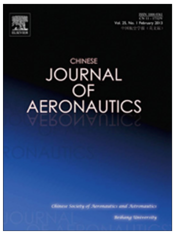

REVIEW ARTICLE

Comparative study between crack closure model and Willenborg model for fatigue prediction under overload effects

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Abstract A comparative study is performed between a crack closure model and the Willenborg model, which can calculate the fatigue crack growth rate under the overload effects. The modified virtual crack annealing (VCA) model is briefly reviewed, which is based on the equivalent plastic zone concept. In this method, the retardation phenomenon is explained by the crack closure level variation, which is derived from the interactions between forward and reverse plastic zones ahead of the crack tip. As a comparison, the Forman equation in conjunction with the Willenborg model is also reviewed. The retardation phenomenon is described by directly modifying the stress intensity factor. It is known that the large plastic zone created by the overload can decelerate the fatigue crack growth rate until the crack grows beyond this region. A relationship between the plastic zone and the modified stress intensity factor is developed, which is a mathematical fitting equation instead of physical-based formulation. The experimental data in aluminum alloys are used to validate these two models. Overall, good agreement is observed between the model predictions and the testing data. It is noted that the approach based on modified VCA model can give more accurate prediction curves than the Willenborg model.

1. Introduction

For many aircraft structures, understanding the fatigue crack growth under service loading conditions is of great significance. Many structure components are frequently subjected to constant amplitude loading with occasional high peak loads, which are called overloads. Due to the constant air current and occasional turbulence during the flight, the fatigue crack growth of the aircraft structural component is always under the influence of overloads. It is necessary to

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deeply understand and precisely predict the fatigue crack growth behavior under the constant amplitude loading with overloads. And many experimental, theoretical and numerical studies are conducted to investigate it.[1–3] Additionally, the retardation phenomenon caused by the overload is a typical load sequence interaction effect.[4,5] The intensive study on overload effect is a prerequisite for theoretically solving the nonlinear crack propagation problem under variable amplitude loading.

Plentiful models were proposed to quantify the nonlinear fatigue crack growth with the overload effect.[6–8] These models could be divided into two main categories: the crack-closure-based models and the plasticity-based models. For the crack-closure-based model, the crack closure level increases as a result of overload, which retards the subsequent fatigue crack growth. Elber[9] introduced the crack closure concept firstly and developed the relationship between the crack growth rate and the effective intensity factor range. In his study, this approach is only validated under the constant amplitude loading. And then, many modified models are proposed to analyze the crack closure variation under overload effects.[10–14] However, in most of the existing models, the crack closure is evaluated either by the finite element analysis or by the indirect experimental measurements. The former is relatively time consuming due to involving the highly nonlinear analysis of cyclic plasticity and contact, while the latter is not suitable for variable amplitude loading.[15,16] Zhang and Liu[17,18] preformed an in-situ scanning electron microscope (SEM) experiment and directly observed the crack closure phenomenon. The virtual crack annealing (VCA) model is developed, which is inspired from this experiment observation. In this paper, this model is modified by introducing the equivalent plastic zone concept which could quantify the previous load sequence effects. For the second category, the methods are based on the assumption that the large monotonic plastic zone ahead of the crack tip leads to the crack growth retardation phenomenon. In these models, the plastic zone size is considered as a modification factor of the fatigue crack growth rate. For example, Wheeler[19] proposed a series of empirical models to evaluate the overload effects. And Willenborg et al.[20] modified these models, which do not incorporate any meaningless parameter. Willenborg et al. described the retardation phenomenon by proposing a relationship between the effective stress intensity factor and the plastic zone size, which is a mathematical fitting equation instead of physical-based formulation. Additionally, this model presents the limitation of estimating the crack arrest for overloads with magnitude $R_{\mathrm{OL}} \geqslant 2$ and the load sequence effects owing to underloads.[21]

This investigation aims to compare the Willenborg approach with the modified VCA model for fatigue crack growth prediction under the constant amplitude loading with overload effects. The paper is organized as follows. First, the modified VCA model and the Willenborg model are reviewed. Then, the fatigue testing data of D16 aluminum alloy and Al 7075-T6 under constant amplitude loading with/without overloads are employed to validate these two models. Finally, the comparative analysis and some conclusions are given based on the current investigation.

2. Methodology

2.1. Modified virtual crack annealing model

Elber[9] introduced the crack closure concept firstly and developed the relationship between the fatigue crack growth rate and the effective stress intensity factor, which can be expressed as

$$\begin{aligned} &\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K_{\mathrm{eff}})^{\mathrm{m}} \\ &\Delta K_{\mathrm{eff}} = K_{\max} - K_{\mathrm{op}} \end{aligned} \tag{1}$$

where $\mathrm{d}a/\mathrm{d}N$ is the crack growth rate; ΔK_{eff} is the effective stress intensity factor; $K_{\max}$ is the stress intensity factor of the peak load; K_{op} is the stress intensity factor of crack closure level; C and m are calibration parameters.

Since the fatigue crack growth is significantly influenced by plasticity ahead of the crack tip, the load sequence effect can be correlated with the plastic deformation in the vicinity of the crack tip. In order to investigate this effect, the plastic state caused by the previous loads should be traced. Therefore, the equivalent plastic zone concept is proposed herein, and the general expression can be written as

$$a_0+\sum_{j=1}^{i}\mathrm{d}a_j+D_{\mathrm{eq},i}=\max\left(a_0+\sum_{j=1}^{i}\mathrm{d}a_j+d_i, a_0+\sum_{j=1}^{i-1}\mathrm{d}a_j+D_{\mathrm{eq},i-1}\right) \tag{2}$$

where $D_{\mathrm{eq},i}$ is the equivalent plastic zone size in the ith cycle; a_0 the initial crack length; $\mathrm{d}a$ the crack increment; d_i the current plastic zone size in the ith cycle; $a_0 + \sum_{j=1}^{i}\mathrm{d}a_j$ the crack length in the ith cycle; i the current cycle number. A schematic sketch is given to illustrate the equivalent plastic zone concept. The loading sequential process and the corresponding plastic state variation are shown in Fig. 1. The dashed zigzag lines represent the loading history. The large plastic zones have been formed at t_1, and the crack tip is O_1 at that moment. The monotonic and reverse plastic zones can be expressed as[10]

$$\begin{cases} d_{\mathrm{m}} = \frac{\pi}{8}\left(\frac{K_{\max}}{\sigma_{\mathrm{y}}}\right)^2 \\ d_{\mathrm{r}} = \frac{\pi}{8}\left(\frac{K_{\max}-K_{\mathrm{op}}}{2\sigma_{\mathrm{y}}}\right)^2 \end{cases} \tag{3}$$

where d_{m} is the monotonic plastic zone size; d_{r} the reverse plastic zone size; σ_{y} is tensile yield strength. The current load is applied at t_2 and the new crack tip is O_2. The large forward and reverse plastic zones, which are the dotted ellipses, form during the largest load cycle in the previous loading history. Before t_2, the following plastic zones do not reach their boundaries respectively even though the crack grows. The solid ellipses represent the equivalent plastic zones ahead of the crack tip O_2. In addition, the actual contour of plastic zone is butterfly-shape instead of circle, but theoretically their diameters along the crack direction are identical (Fig. 1). In current study, the plastic zone effects are calculated by the equivalent plastic zone which is considered to be in direct proportion to the circular diametric distance. This proportional relation is indicated by the geometry modification factor which is equal

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