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Orbital reference frame estimation with power spectral density constraints for drag-free satellites

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Abstract The drag-free satellites are widely used in the field of fundamental science as they enable the high-precision measurement in pure gravity fields. This paper investigates the estimation of local orbital reference frame (LORF) for drag-free satellites. An approach, taking account of the combination of the minimum estimation error and power spectral density (PSD) constraint in frequency domain, is proposed. Firstly, the relationship between eigenvalues of estimator and transfer function is built to analyze the suppression and amplification effect on input signals and obtain the eigenvalue range. Secondly, an optimization model for state estimator design with minimum estimation error in time domain and PSD constraint in frequency domain is established. It is solved by the sequential quadratic programming (SQP) algorithm. Finally, the orbital reference frame estimation of low-earth-orbit satellite is taken as an example, and the estimator of minimum variance with PSD constraint is designed and analyzed using the method proposed in this paper.

1. Introduction

Drag-free satellites can eliminate the non-gravitational accelerations, i.e., solar radiation pressure and atmospheric drag to obtain a free-falling dynamical environment for the floating proof-mass inside the satellite. This kind of satellite is widely used in the measurement of Earth gravity field, the test of Equivalence Principle and the detection of gravitational waves.[1] Both NASA and ESA have successfully launched drag-free satellites to measure the Earth gravity[2–4] and proposed the next generation missions, for instance, the NGGM mission[5,6] and the GRACE follow-on mission.[7,8] The Microscope mission[9] for testing the equivalence principle, and LISA mission[10,11] for detecting gravitational waves have also been scheduled. China has also proposed several space exploration plans based on drag-free technology.[12,13]

Drag-free concept was originally proposed by Pugh,[14] and then studied systematically by Lange.[15] With the development of drag-free missions, a wide variety of studies about the drag-free control has been carried out.[16–19] The control system con-

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sists of the inner and outer control loops which take charge of non-gravitational accelerations and attitude pointing, respectively.[20] The satellite attitude measured by star sensor in earth inertial frame should be aligned with the local orbital reference frame (LORF) which is determined by the satellite position and velocity. The measured attitude is transformed into LORF as the input to the control loop. Due to the stringent requirements on the power spectral density (PSD) of Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) mission, that is, the residual linear non-gravitational accelerations with the PSD below $10^{-6}\,\mathrm{m/s^2/Hz^{1/2}}$, as well as the angular accelerations with the PSD below $10^{-6}\,\mathrm{rad/s^2/Hz^{1/2}}$ in measurement bandwidth (MBW), the attitude control error of the drag-free satellite is required to reach the level of $10^{-5}\,\mathrm{rad/Hz^{1/2}}$ in terms of PSD.[20] Thus, the knowledge of LORF should be estimated at a level better than attitude accuracy in order to minimize the contribution to the overall error budget.[21] Simultaneously, the high-precision state estimation or accurate measurement of inter-satellites range is the prerequisite of the recovery of gravitational field or the extraction of signals of gravitational waves. Finally, the state estimation is critical for the success of this kind of drag-free satellites and related space science missions.

The satellite position and velocity are measured by onboard GPS, which introduces noises to be filtered. For the state estimation, a series of nonlinear estimators has been proposed in the past. Most of them are nonlinear extensions of the Kalman Filter[22] which include the Extended Kalman Filter,[23] Unscented Kalman Filter,[24] and others. These filters are developed for minimum estimation error and require the knowledge of input noise statistics. However, the drag-free satellite whose main function is the high-precision measurement within certain MBW has special constraints in frequency domain for state estimation. The PSD of estimation error is strictly suppressed below the specification in MBW, but can be increased at the frequency lower than left bound of MBW since the peak attitude is allowed to relax to mrad level. These filters based on optimal estimation error are unable to cover the frequency domain constraints. Thus, a new type of estimator, which has the capability of combining both the PSD constraint in frequency domain and minimum estimation error in time domain, needs to be investigated.

For the attitude estimation, Evers[16] analyzed the dynamics and estimated the colored measurement noises for GOCE satellite by tuning the covariance matrix of model noises. Canuto[25–27] proposed the embedded model to deal with the state estimation and drag-free control. For orbital reference estimation, the embedded model consists of high-precision discrete orbital dynamics and disturbed dynamics models. An error feedback gain was designed to construct the time invariant estimator. The PSD constraint of estimation error is met by tuning the eigenvalues of state estimator. Moreover, it doesn't rely on the statistic knowledge of input error.

In this paper, an extensive study about estimator design is carried out on the basis of the work in Refs.[21,24] A design method for optimal LORF estimation is proposed to realize the minimum estimation variance with PSD constraints for drag-free satellites. First, the linear time-invariant estimator is established by the time-varying gain matrix. The PSD of input and output signals is connected by the transfer function of the estimator. Second, the PSD constraint is mapped to the requirements on eigenvalues and transfer function by analyzing the magnification and suppression effect on the PSD of input signals. The optimization issue, the minimum estimation error with the PSD constraints, is modeled. It is solved by a local optimal algorithm, that is, the sequential quadratic programming (SQP) algorithm. To validate the adaptability of this method, the LORF of a 250 km low-earth-orbit (LEO) satellite is estimated by the method proposed in this paper. This method is also suitable for the orbital reference estimation of all LEO drag-free satellites. Furthermore, it provides the technology support for the future satellite gravity measurement mission, Space Advanced Gravity Measurement (SAGM),[12] in China.

2. Background of orbit estimation

2.1. Equations of orbital movement

It is assumed that the drag-free satellite can totally eliminate the non-gravitational accelerations, and then the satellite moves in a pure gravitational field. The acceleration of gravity with J_2 perturbation is

$$\boldsymbol{g} = \frac{\mu}{r^3}\left\{\boldsymbol{I} - \frac{3}{2}J_2\left(\frac{R}{r}\right)^2\left[5\left(\frac{z}{r}\right)^2\boldsymbol{I} - \boldsymbol{\Gamma}\right]\right\}\boldsymbol{r} + \delta\boldsymbol{g} \tag{1}$$

where J_2 is set to be 1.08×10^{-3}; R is the Earth equator radius, and $R = 6378.14$ km in this paper; $\boldsymbol{r}$ denotes satellite vector, $r = \|\boldsymbol{r}\|$; $\boldsymbol{\Gamma} = \mathrm{diag}(1, 1, 3)$; μ is the gravitational constant of the Earth; $\delta\boldsymbol{g}$ is the residual gravity acceleration; z is the third coordinate component of satellite state.

The orbit of satellite is not circular due to the non-spherical perturbations. Thus, r in Eq. (1) will be time-varying. If the mean orbit height is h, the 1st order expansion of Eq. (1) at the mean radius $\underline{r} = R + h$ is

$$\begin{cases} \boldsymbol{g}(\boldsymbol{r}(t)) = -\underline{\omega}_0^2(\boldsymbol{I} + \partial\boldsymbol{\Omega}(\boldsymbol{r}))\boldsymbol{r}(t) \\ \partial\boldsymbol{\Omega}(\boldsymbol{r}) = 3(1 - \underline{\boldsymbol{r}}^{\mathrm{T}}\boldsymbol{r}/\underline{r}^2)I + \gamma_0(z/\underline{r})^2\boldsymbol{I} - \boldsymbol{\Gamma}_1 \\ \gamma_0 = \frac{15}{2}J_2(R/\underline{r})^2 \\ \boldsymbol{\Gamma}_1 = \frac{3}{2}J_2(R/\underline{r})^2\boldsymbol{\Gamma} \end{cases} \tag{2}$$

where $\underline{\omega}_0$ is the mean angular velocity.

Hence, the differential equation of orbit state $[\boldsymbol{r}, \boldsymbol{v}]^{\mathrm{T}}$ can be written as

$$\begin{bmatrix} \dot{\boldsymbol{r}} \\ \dot{\boldsymbol{v}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\underline{\omega}_0^2(\boldsymbol{I} + \partial\boldsymbol{\Omega}(\boldsymbol{r})) & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{r} \\ \boldsymbol{v} \end{bmatrix}(t) + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{bmatrix}\boldsymbol{a}_{\mathrm{d}}(t) \tag{3}$$

where $\boldsymbol{a}_{\mathrm{d}}$ is the sum of residual non-gravitational accelerations and gravity anomalies. The J_2 term is included in $\partial\boldsymbol{\Omega}(\boldsymbol{r})$.

2.2. Discrete-time dynamics

2.2.1. Simplification of discrete dynamics

The time interval of drag-free control is 0.1 s which is far less than the orbital period. Thus, the differential Eq. (3) varies slowly during the 0.1 s. The angular rate within this time interval could be approximated as

$$-\underline{\omega}_0^2(\boldsymbol{I} + \partial\boldsymbol{\Omega}(\boldsymbol{r}_i)) = -\boldsymbol{\omega}_i^2 \tag{4}$$

where $\boldsymbol{r}_i$ and $\boldsymbol{\omega}_i$ are the position vector and angular velocity vector of satellite at time t_i, respectively.