Chinese Journal of Aeronautics, (2016), xxx(xx): xxx–xxx

Chinese Society of Aeronautics and Astronautics
& Beihang University

Chinese Journal of Aeronautics

cja@buaa.edu.cn
www.sciencedirect.com

FULL LENGTH ARTICLE

# Probability hypothesis density filter with adaptive parameter estimation for tracking multiple maneuvering targets

**Yang Jinlong** [a,b,*], **Yang Le** [a], **Yuan Yunhao** [a], **Ge Hongwei** [a]

[a] *School of Internet of Things Engineering, Jiangnan University, Wuxi 214122, China*
[b] *Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), Wuxi 214122, China*

Received 31 December 2015; revised 22 September 2016; accepted 22 September 2016

**KEYWORDS**

Adaptive parameter estimation;
Multiple target tracking;
Multivariate Gaussian distribution;
Particle filter;
Probability hypothesis density

**Abstract** The probability hypothesis density (PHD) filter has been recognized as a promising technique for tracking an unknown number of targets. The performance of the PHD filter, however, is sensitive to the available knowledge on model parameters such as the measurement noise variance and those associated with the changes in the maneuvering target trajectories. If these parameters are unknown in advance, the tracking performance may degrade greatly. To address this aspect, this paper proposes to incorporate the adaptive parameter estimation (APE) method in the PHD filter so that the model parameters, which may be static and/or time-varying, can be estimated jointly with target states. The resulting APE-PHD algorithm is implemented using the particle filter (PF), which leads to the PF-APE-PHD filter. Simulations show that the newly proposed algorithm can correctly identify the unknown measurement noise variances, and it is capable of tracking multiple maneuvering targets with abrupt changing parameters in a more robust manner, compared to the multi-model approaches.



## 1. Introduction

Multiple target tracking (MTT) has gained wide attentions due to its theoretical and practical importance. Conventionally, the MTT problem was tackled from the perspective of data association. A number of tracking algorithms were developed in the literature on the basis of techniques including the joint probabilistic data association (JPDA),[1] joint integrated probabilistic data association (JIPDA)[2] and multiple hypothesis tracking (MHT).[3] These methods are generally computationally intensive and some of them even have exponentially growing complexity as the target number increases. Reduced-

* Corresponding author at: School of Internet of Things Engineering, Jiangnan University, Wuxi 214122, China.
E-mail address: yjlgedeng@163.com (J. Yang).
Peer review under responsibility of Editorial Committee of CJA.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.cja.2016.09.010
1000-9361 

complexity techniques were proposed in Refs. 4–6. They are better for real-time applications at the cost of degraded estimation accuracy.

Recently, the use of the random finite set (RFS) theory[7–11] attracted great interests, because it provides an elegant formulation of the MTT problem. But the obtained multi-target Bayesian filter is intractable in most practical scenarios due to the inherent combinatorial nature of multi-target state densities and the need for evaluating set integrals over high dimensional spaces. To deal with the intractability, the probability hypothesis density (PHD) filter[7] and the cardinalized PHD (CPHD) filter[8] were developed using the first-order moment and cardinality distributions. Existing closed-form realizations of PHD filters include the particle filter PHD (PF-PHD),[9,10] Gaussian mixture PHD (GM-PHD) filter[11] and various modified versions.[12–15] Different from the PHD and CPHD filters, the cardinality-balanced multi-target multi-Bernoulli (CBMeMBer) filter was proposed in Ref. 16 for MTT by directly propagating the approximate posterior density of the targets. These algorithms exhibit good performance only when the model parameters, such as the measurement noise variances, are known precisely. In the presence of unknown time-varying measurement noise variances, the variational Bayesian (VB) approximation method[17–19] can be employed to recursively estimate the joint PHDs of the multi-target states and the measurement noise variance.[20,21] However, these methods may suffer from performance degradation if targets manoeuver with unknown abruptly changing parameters.

For maneuvering target tracking, the use of the jump Markov system (JMS) that switches among a set of candidate models in a Markovian fashion has proved to be effective.[22,23] Pasha et al.[24] introduced the linear JMS into PHD filters and derived a closed-form solution for the PHD recursion. Furthermore, the unscented transform (UT) and the linear fractional transformation (LFT) were combined with the closed-form solution for the nonlinear jump Markov multi-target models in Refs. 25, 26 In Ref. 27, a GM-PHD filter for jump Markov models was developed by employing the best-fitting Gaussian (BFG) approximation approach. These algorithms assume the Gaussianity of the PHD distribution, which may limit their application scope. The multiple-model particle PHD (MMP-PHD) filter, the MMP-CPHD filter and MMP-CBMeMBer filter are implemented by using the sequential Monte Carlo (SMC) method and their improved versions were presented in Refs. 28–30 Most of the MM-based filters track multiple maneuvering targets through the interaction of multiple models, which is realized via combining estimates from different models according to their respective model likelihoods. The difficulty of applying them in tracking targets with abruptly changing maneuvering parameters comes from the need to specify a prior set of candidate models. In other words, they may suffer from the curse of dimensionality: if we wish to account for multiple unknown parameters, the number of models needed would increase exponentially with the number of parameters.

In this work, we incorporate the adaptive parameter estimation (APE) technique into the PHD filter for addressing the problem of multiple maneuvering target tracking, where both static and time varying unknown parameters, namely the measurement noise variance and the parameters associated with abrupt target maneuvers, are presented and need to be estimated. The inverse Gamma (IG) distribution is used to approximate the posterior distribution of the measurement noise variances while the adaptive Liu and West (LW) filter is adopted to propagate the posterior marginal of the time-varying parameters as a mixture of multivariate Gaussian distributions.[31–33] The obtained APE-PHD filter is realized using the particle filter (PF), which leads to the PF-APE-PHD algorithm for tracking multiple maneuvering targets in the presence of unknown model parameters. Simulation results show that the proposed algorithm exhibits better robustness and improved tracking performance over the MM-PHD and MM-CPHD algorithms.

The remainder of this paper is organized as follows. Section 2 formulates the problem of tracking a target in the presence of unknown model parameters. It also briefly reviews the APE technique and the PHD filter. Section 3 develops the APE-PHD algorithm and presents the closed-form solution, the PF-APE-PHD algorithm. Simulation results are given in Section 4. Finally, conclusions are provided in Section 5.

## 2. Preliminary

### 2.1. Problem formulation

The state-space model for tracking a single target moving on a two-dimensional plane is given by

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}\boldsymbol{x}_k + \boldsymbol{G}\boldsymbol{v}_k \tag{1}$$

$$\boldsymbol{y}_k = h(\boldsymbol{x}_k) + \boldsymbol{w}_k \tag{2}$$

where $\boldsymbol{x}_k = [x_k, v_{x_k}, y_k, v_{y_k}]^{\mathrm{T}}$ denotes the target state at time $k$, $(x_k, y_k)$ and $(v_{x_k}, v_{y_k})$ denote its position and velocity. $\boldsymbol{F}$ and $\boldsymbol{G}$ are the state transition matrix and the process noise gain matrix. $\boldsymbol{y}_k$ is the measurement vector. $\boldsymbol{v}_k$ and $\boldsymbol{w}_k$ denote the process noise and the measurement noise. They are independent of each other and modeled as zero-mean Gaussian random processes with covariance $\boldsymbol{Q}_k$ and $\boldsymbol{R}_k$.

In many practical applications, the state-space model in Eqs. (1) and (2) may contain unknown parameters. For example, if the target conducts a coordinated turn (CT),[28] the state transition matrix would become

$$\boldsymbol{F}(\omega) = \begin{bmatrix} 1 & \frac{\sin\omega T}{\omega} & 0 & -\frac{1-\cos\omega T}{\omega} \\ 0 & \cos\omega T & 0 & -\sin\omega T \\ 0 & \frac{1-\cos\omega T}{\omega} & 1 & \frac{\sin\omega T}{\omega} \\ 0 & \sin\omega T & 0 & \cos\omega T \end{bmatrix} \tag{3}$$

The turn rate $\omega$ may be unknown and time-varying. Besides, the measurement noise covariance $\boldsymbol{R}_k$ may also be unknown. In these scenarios, we need to jointly estimate the posterior distribution of the target states and the unknown parameters from the measurements.

Let $\boldsymbol{\Phi}_k$ be a column vector that collects the static and time-varying parameters in the state-space model. The posterior probability density function (PDF) of the target state vector $\boldsymbol{x}_k$ and $\boldsymbol{\Phi}_k$ conditioned on the measurements up to time $k$ is, according to Bayes' rule,

$$p(\boldsymbol{x}_k, \boldsymbol{\Phi}_k|\boldsymbol{y}_{1:k}) = \frac{p(\boldsymbol{y}_k|\boldsymbol{x}_k, \boldsymbol{\Phi}_k)p(\boldsymbol{x}_k, \boldsymbol{\Phi}_k|\boldsymbol{y}_{1:k-1})}{\iint p(\boldsymbol{y}_k|\boldsymbol{x}_k, \boldsymbol{\Phi}_k)p(\boldsymbol{x}_k, \boldsymbol{\Phi}_k|\boldsymbol{y}_{1:k-1})\mathrm{d}\boldsymbol{x}_k\mathrm{d}\boldsymbol{\Phi}_k} \tag{4}$$

where $p(\boldsymbol{x}_k, \boldsymbol{\Phi}_k|\boldsymbol{y}_{1:k-1})$ is the predicted PDF given by