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Variational Bayesian labeled multi-Bernoulli filter with unknown sensor noise statistics

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Abstract It is difficult to build accurate model for measurement noise covariance in complex backgrounds. For the scenarios of unknown sensor noise variances, an adaptive multi-target tracking algorithm based on labeled random finite set and variational Bayesian (VB) approximation is proposed. The variational approximation technique is introduced to the labeled multi-Bernoulli (LMB) filter to jointly estimate the states of targets and sensor noise variances. Simulation results show that the proposed method can give unbiased estimation of cardinality and has better performance than the VB probability hypothesis density (VB-PHD) filter and the VB cardinality balanced multi-target multi-Bernoulli (VB-CBMeMBer) filter in harsh situations. The simulations also confirm the robustness of the proposed method against the time-varying noise variances. The computational complexity of proposed method is higher than the VB-PHD and VB-CBMeMBer in extreme cases, while the mean execution times of the three methods are close when targets are well separated.

1. Introduction

Random finite set (RFS) theory[1,2] provides a systematic mathematical foundation for the multi-target tracking (MTT) problem and arises widespread interest in the last decade. Major algorithms include probability hypothesis density (PHD) filter[2], cardinalized PHD (CPHD) filter[3,4] and cardinality balanced multi-target multi-Bernoulli (CBMeMBer) filter.[5] Strictly, those RFS-based filters are not multi-target tracker because they assume that the objects are indistinguishable.[6] For this problem, Ref.[7] proposed a newly labeled RFS approach, known as the δ-generalized labeled multi-Bernoulli (δ-GLMB) filter. The main advantages of the δ-GLMB filter over traditional RFS filters are that it can output trajectories and has a better performance in harsh environments, such as low detection probability and large sensor noise variances, though with higher computation burden. The labeled multi-Bernoulli (LMB) filter[8] is proposed to reduce the number of components of δ-GLMB by moment approximation and track grouping strategy.

The standard RFS algorithms mentioned above hold the assumption that the prior knowledge of sensor noise statistics are known, while it is usually difficult to build an accurate model for the sensor noise variances in practice. Multiple model (MM)[9] and particle filter (PF)[10] are adaptive Bayesian methods for unknown parameters. However, they have been

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computationally expensive since their performances rest on the number of models or particles. Compared to MM and PF methods, variational Bayesian (VB) approximation[11,12] is more dedicate and efficient in dealing with uncertain sensor noise variances tracking problems within linear Gaussian condition. Yang,[13] Zhang[14] and Wu et al.[15,16] extended the VB method to MTT scenario with the PHD filter, respectively. Adaptive VB methods based on the CBMeMBer filter have been studied[17–19] as well. Due to the inherent weakness of PHD and CBMeMBer, the VB-PHD and VB-CBMeMBer filters do not perform well in the low signal-to-noise ratio (SNR) situations. To improve this problem, this contribution introduces the VB approach to the LMB filter framework, making it suitable for real world scenarios. The proposed VB-LMB filter inherits the efficiency of VB approximation and the advantages of LMB filter in harsh environments, as well as the higher computational price.

This manuscript is organized as follows. Section 2 briefly discusses the background of the VB approximation and standard LMB filter. The VB-LMB filter and its implementation issues are detailed in Section 3. Section 4 shows the results of the proposed and compared algorithms. Finally, conclusions can be found in Section 5.

2. Background

2.1. VB approximation

The dynamic model of linear Gaussian system is given by

$$\boldsymbol{x}_+ = \boldsymbol{F}\boldsymbol{x} + \boldsymbol{v} \tag{1}$$

$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{w} \tag{2}$$

where $\boldsymbol{x}$ is the target state and $\boldsymbol{z}$ the measurement; $\boldsymbol{F}$ and $\boldsymbol{H}$ represent the transition matrix and observation matrix, respectively, $\boldsymbol{v}$ and $\boldsymbol{w}$ are independent zero-mean Gaussian noises with covariance $\boldsymbol{Q}$ and $\boldsymbol{R}$, respectively. $\boldsymbol{R}$ is supposed to have the form of $\boldsymbol{R} = \mathrm{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_d^2)$, where σ_i^2 $(i = 1, 2, \ldots, d)$ is the measurement noise variance and d the dimension of measurement. $+$ denotes the next time step.

The goal of the variational approximation is to jointly estimate the target state and measurement noise covariance. Suppose that the joint prior distribution has the factored form of

$$p(\boldsymbol{x}, \boldsymbol{R}) = \mathcal{N}(\boldsymbol{x}; \boldsymbol{m}, \boldsymbol{P}) \prod_{j=1}^{d} \mathcal{IG}(\sigma_j^2; \alpha_j, \beta_j) \tag{3}$$

where $\mathcal{N}(\cdot)$ and $\mathcal{IG}(\cdot)$ represent the Gaussian and inverse Gamma distributions, respectively. As the assumption that target state and measurement noise covariance are mutually independent, at prediction step, the Gaussian and inverse Gamma distributions in Eq. (3) evolve independently. Thus, the predictive distribution keeps the form of the prior distribution

$$p_+(\boldsymbol{x}, \boldsymbol{R}) = \mathcal{N}(\boldsymbol{x}; \boldsymbol{m}_+, \boldsymbol{P}_+) \prod_{j=1}^{d} \mathcal{IG}(\sigma_j^2; \alpha_{+,j}, \beta_{+,j}) \tag{4}$$

To make the coupled update step tractable, the VB method has been applied to obtaining the posterior distribution $p(\boldsymbol{x}, \boldsymbol{R}|z)$. Let

$$\tilde{p}(\boldsymbol{x}, \boldsymbol{R}|z) = Q_{\boldsymbol{x}}(\boldsymbol{x}) Q_{\boldsymbol{R}}(\boldsymbol{R}) \tag{5}$$

and the Kullback-Leibler (KL) divergence from $\tilde{p}(\boldsymbol{x}, \boldsymbol{R}|z)$ to $p(\boldsymbol{x}, \boldsymbol{R}|z)$ is given by

$$\mathrm{KL}[\tilde{p}(\boldsymbol{x}, \boldsymbol{R}|z) || p(\boldsymbol{x}, \boldsymbol{R}|z)] = \int \tilde{p}(\boldsymbol{x}, \boldsymbol{R}|z) \ln \frac{\tilde{p}(\boldsymbol{x}, \boldsymbol{R}|z)}{p(\boldsymbol{x}, \boldsymbol{R}|z)} \mathrm{d}\boldsymbol{x}\mathrm{d}\boldsymbol{R} \tag{6}$$

The KL divergence will achieve minimum if

$$Q_{\boldsymbol{x}}(\boldsymbol{x}) \propto \exp\{E_{Q_{\boldsymbol{R}}(\boldsymbol{R})}[\ln(p(\boldsymbol{x}, \boldsymbol{R}, z))]\} \tag{7}$$

$$Q_{\boldsymbol{R}}(\boldsymbol{R}) \propto \exp\{E_{Q_{\boldsymbol{x}}(\boldsymbol{x})}[\ln(p(\boldsymbol{x}, \boldsymbol{R}, z))]\} \tag{8}$$

The results in Ref.[12] illustrate that $Q_{\boldsymbol{x}}(\boldsymbol{x})$ has a Gaussian distribution and $Q_{\boldsymbol{R}}(\boldsymbol{R})$ is a product of inverse Gamma distributions. Consequently, the approximated posterior distribution has the same factored form as the prior distribution.

2.2. Original LMB filter

The traditional RFS-based filters assume that targets are indistinguishable. Recently, Ref.[7] introduced a labeled model to address the uniqueness of individual target. Let X be a finite set that represents the multi-target state and the labeled RFS is described as

$$X = \{(\boldsymbol{x}, l)_i\} \quad i = 1, 2, \ldots, M \tag{9}$$

where $\boldsymbol{x}$ and $l \in \boldsymbol{L}$ are the state and label of a single target, respectively, and $\boldsymbol{L}$ is the label space; M is the number of targets. Let the notation $\Delta(X) = \delta_{|X|}(|\mathcal{L}(X)|)$ be the distinct label indicator, where $\mathcal{L}(X)$ denotes the labels set of X; $|\cdot|$ indicates the cardinality of a set.

An important form of multi-target density called d-GLMB RFS is introduced, also known as Vo-Vo density[7]

$$\pi(X) = \Delta(X) \sum_{I \in \mathcal{F}(\boldsymbol{L})} \omega(I) \delta_I(\mathcal{L}(X)) [p]^X \tag{10}$$

where each $I \in \mathcal{F}(\boldsymbol{L})$ represents a hypothesis with a set of track labels, and $\mathcal{F}(\boldsymbol{L})$ is a subset collection of space $\boldsymbol{L}$; $\omega(I)$ is the weight of hypothesis I and $\sum_{I \in \mathcal{F}(\boldsymbol{L})} \omega(I) = 1$; $p(\boldsymbol{x}, l)$ denotes the spatial distribution of target; the unlabeled multi-target exponential function is defined as $[p]^X = 1$ when $X = \varnothing$ and when $X \neq \varnothing$, $[p]^X = \prod_{\boldsymbol{x} \in X} p(\boldsymbol{x})$.

The δ-GLMB RFS is a conjugate prior of the Chapman-Kolmogorov equation and Bayes multi-target inference, which facilitates a close form implementation of the multi-target Bayesian filter. The optimal solution to the δ-GLMB filter is intractable since the number of components grows exponentially. The LMB filter is an approximated version of the δ-GLMB filter. This section outlines the prediction and update of the LMB method, and the full details can be found in Ref.[8].

Suppose that the prior multi-target density can be described as LMB density

$$\pi(X) = \{r^{(l)}, p^{(l)}(\boldsymbol{x})\}_{l \in \boldsymbol{L}} \tag{11}$$

where $r^{(l)}$ and $p^{(l)}(\boldsymbol{x})$ are the existence probability and the spatial distribution of a target, respectively. Assume that the density of new birth is $\{p_{\mathrm{B}}^{(l)}, p_{\mathrm{B}}^{(l)}(\boldsymbol{x})\}_{l \in \boldsymbol{B}}$, where $\boldsymbol{B}$ is the label space of birth and $\boldsymbol{L} \cap \boldsymbol{B} = \varnothing$, then the LMB predictive density $\pi_+(X)$ is given by

$$\pi_+(X) = \{r_{\mathrm{S}}^{(l)}, p_{\mathrm{S}}^{(l)}(\boldsymbol{x})\}_{l \in \boldsymbol{L}} \cup \{r_{\mathrm{B}}^{(l)}, p_{\mathrm{B}}^{(l)}(\boldsymbol{x})\}_{l \in \boldsymbol{B}} \tag{12}$$