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Research paper

Nonlinear dynamics of displaced non-Keplerian orbits with low-thrust propulsion

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ABSTRACT

This paper discusses the stability, transition and control of displaced non-Keplerian orbits by the spacecraft using low-thrust propulsion. The two-body dynamical model developed in the polar coordinates is parameterized by the thrust pitch angle, and then two of the hyperbolic and elliptic equilibria are solved from it. The bounded motions near two equilibria are investigated by dynamical system techniques to find out all the stable and unstable periodic trajectories, and two scenarios of the resonant periodic trajectory are presented. Regardless of the thrust pitch angle, all the transit orbits are numerically demonstrated to be restricted inside the invariant manifolds of Lyapunov orbit near the hyperbolic equilibrium. Then the transit orbits can be distinguished from non-transit ones by the restriction of three-dimensional invariant manifolds projected onto the Poincaré section or position space. Based on the influence of thrust direction on the system topology, operating the thrust pitch angle is an effective tool to achieve the transfer within different types of KAM tori, or even transfer beyond the KAM tori.

1. Introduction

The concept of counter-acting gravity through a thrust vector was introduced by Dusek [1], who noted that a spacecraft could be kept in a stable orbit around the collinear points under the favorable effects of gravitational forces combined with the thrust application. Especially, the use of continuous thrust perpendicular to the flight direction can force the spacecraft out of a natural orbit into a displaced non-Keplerian orbit (NKO) [2]. Due to the superiority in applications involving in-situ observations [3], displaced orbits whose orbital plane do not contain the primary's center-of-mass, have been of great interest. In particular, since changes to the arctic ice pack open navigation channels for shipping, inserting the spacecraft into a non-Keplerian displaced orbit over the Earth [4–6] for continuous observation and relay communication of the Earth north polar region will be a key issue in the future. Benefiting from the continuous thruster and ultra-high specific impulse of advanced propulsion systems [7–8], the planning of displaced orbits with low-thrust propulsion can be considered

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Nomenclature

ρ	orbit radius
z	displacement distance parallel to the sun line
κ	magnitude of the scaled thrust acceleration
α	thrust pitch angle
h_z	constant angular momentum
r	distance between the Earth and spacecraft
U	potential function
E	system energy
L^u	hyperbolic equilibrium
L^s	elliptic equilibrium

a feasible option of scientific space missions. In addition, displaced NKOs have a diverse range of potential applications in both the Sun-centered and planet-centered studies, such as the solar physics and an Earth synchronous orbit for continuous observations and space weather monitoring [9], displaced orbits above Earth-Moon L_2 libration point for lunar far-side communication [10], and a new family of NKOs displaced above or below the Earth's equatorial plane for the increasing number of available slots for geostationary communications satellites [11].

McInnes [12] investigated the existence, stability, control and application of displaced NKOs with an additional thrust-induced acceleration in the two-body problem, where the problem is parameterised by the orbit period which generates some new families of orbits. Xu and Xu [13] achieved a displaced non-Keplerian orbit above a planet by orientating the normal of the solar sail to the Sun line and analysed its nonlinear dynamics and the control problem. Ceriotti and McInnes [14] studied optimal pole-sitter orbits in the restricted three-body problem and proposed a hybrid solar sail combining the solar radiation pressure (SRP) and solar electric propulsion (SEP) to enable a near-term pole-sitter mission. Heiligers et al. [15] proposed the use of hybrid sails to design out-of-plane and in-plane displaced geostationary orbits, which can allow spacecraft to be stationary with respect to their ground station by cancelling the residual in-plane acceleration with the solar electric propulsion thruster. Previous work on this subject has been devoted to investigate the motion of solar sails around displaced orbits in the restricted three-body problem, or non-Keplerian orbits displaced over planetary by low-thrust propulsion with a zero pitch angle, whereas displaced orbit by low-thrust propulsion in arbitrary directions is rarely considered.

Motivated by the driving applications of geocentric displaced orbits, this paper investigates the nonlinear dynamics of non-Keplerian orbits displaced above the Earth with low-thrust propulsion. Building from the dynamical model in Ref. [16], but using the period of the orbit as parameter, this paper introduces the nonzero pitch angle of the spacecraft thrust and discusses how the thrust direction influences the stability, transition and control of displaced NKOs. Instead of focusing on a specific mission, this paper aims to get a general idea of displaced orbit, including all the possible dynamical behaviours (transition, stability, control) which may be used in future applications. Different from thrust direction determining the geometry of the Hill's region, the thrust magnitude just impacts the size of Hill's region, which has less influence on the dynamical motions. Thus, cases of different thrust pitch angle are simulated to show the mechanism and influence, and a suitable thrust magnitude is chosen to show these behaviours in elegant plots.

The remainder of this paper is organized as follows. Considering the two-body dynamical model in the rotating reference frame, two of the hyperbolic and elliptic equilibria parameterized by the thrust are solved and then the basic configurations of Hill's region are presented in Section 2. Then, the bounded motions near the two equilibria are investigated by dynamical system techniques in Sections 3. Next, the stability and transition of trajectories in the neck region are analysed in Section 4 in two ways to distinguish transit orbits from non-transit ones. Finally, according to the influence of the pitch angle on the transition, Section 5 employs the α-control to effectively achieve the orbital transfer within the KAM tori and beyond tori. Just with the change in the thrust direction, the application of the former successfully extends several natural segments to an artificial trajectory and generates a large number of artificial trajectories, and the application of the latter can be viewed as a station-keeping technique to prevent the spacecraft escaping from the unstable motions.

2. Nonlinear dynamics of displaced non-Keplerian orbit

2.1. Orbital dynamics with low-thrust propulsion

In general, displaced NKOs are investigated by considering the dynamics of a spacecraft of mass m at position $\boldsymbol{r}$ in a reference frame $\boldsymbol{R}$ rotating at an angular velocity $\boldsymbol{\omega}$ relative to an inertial frame $\boldsymbol{I}$ [2], as shown in Fig. 1. It is assumed that the spacecraft has active propulsion generating thrust $\boldsymbol{T}$ in the direction $\boldsymbol{n}$. This paper considers the two-body dynamics of a spacecraft in a rotating frame of reference, ignoring the higher harmonics of the gravitational potential and then generates NKOs displaced above the Earth by using low-thrust propulsion with different thrust directions.