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Research paper

Dynamic stability of an SIQS epidemic network and its optimal control

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ABSTRACT

In order to better understand and utilize the quarantine control when encountering outbreaks of infectious diseases, this paper introduces a nonlinear SIQS epidemic model on complex networks. By using complex network theory and Lyapunov function method, we obtain its basic reproduction number and global stability of both disease-free equilibrium and endemic equilibrium. Moreover, we investigate the optimal quarantine control problem for reducing control cost. By applying the optimal control theory, we obtain existence and uniqueness of the optimal control and the model's optimal solution. These results are verified by some numerical examples, and the influence of network structure on the optimal control is also studied.

1. Introduction

It is well known that infectious disease is a major public health threat to human. Due to the heterogenous contacts among individuals in real world, many researches have focused on the spreading of epidemic under complex network framework in the past two decades. The appearance of some famous network models accelerates the investigation of epidemic spreading. The two most famous network models are small-world network [1] and BA network [2]. By embedding these complex network structures into traditional epidemic models, many significant results [3–10] have been obtained in the last two decades.

In order to better understand the spreading process of contagious diseases, epidemic threshold analysis and stability analysis are important from the view of nonlinear science and mathematics. With these theoretical analysis, we can seize the dynamical behaviour of contagious diseases, which can help us propose effective control methods. For examples, utilizing the Lyapunov function method and graph theory, epidemic threshold and the global stability of equilibriums of a class of multi-group SIR epidemic models with varying subpopulation sizes were addressed in Guo et al. [11]. By constructing monotone sequences, Wang and Dai [12] have proven the global stability of the famous SIS epidemic network model established by Pastor–Satorras and Vespignani. In [13], Zhu et al. have applied the stability theory of cooperative systems to investigate the global stability of an epidemic network with adaptive weights. In [14], by using the Routh–Hurwitz stability and Lyapunov

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stability methods, Xu et al. have investigated the global stability of equilibriums of a variation epidemic spreading model on complex networks. By using the linear stability analysis and Lyapunov function method, Zhang and Jin [15] have addressed the stability of an epidemic model with birth and death on networks.

Another important research topic of epidemic networks is how to propose effective method for disease control. From the frequent news reports about avian influenza, ebola virus, etc., we can see that quarantine of infected individuals is a very useful control method to constrain the prevalence [16–18]. The most immediate way to control epidemic spreading may be quarantining infected individuals, as it breaks spatially the connections between infected individuals and susceptible individuals. Especially, before the appearance of relevant vaccines and effective treatment for an infectious disease, quarantine of infected individuals must be the most effective way to control disease spread. In the past two decades, some important methods [19–21] have been introduced to control outbreaks of infectious diseases on networks, such as, targeted immunization, acquaintance immunization, proportional immunization, and so on. As we know, these methods may be infeasible until the corresponding vaccines are developed successfully. So, the theoretical study of quarantine control on epidemic networks is clearly of great importance.

In recent several years, with the great development of complex network theory, the solution of optimal control problem in epidemic networks becomes exigent [22]. Though constant control can realize the control goal (reducing the infected individuals to very low level), it needs great control cost (consuming medical resource) and may be actually unfeasible as the resource of medical treatment is generally limited. So, a feasible strategy is to utilize bounded time-varying control to balance the control cost and control goal. In the past, the applications of optimal control in epidemic dynamics were mainly focused on homogenous contact networks which are described by low-dimension systems [23–28]. In the real world, the human contact networks are usually heterogenous, i.e., there exist great degree differences among the nodes of a network. So, it is a very realistic and challenging topic to solve the optimal control problem in epidemic models on complex networks with heterogenous structures, which are generally described by high-dimension systems. Recently, some interesting works of optimal control of epidemic networks have been reported. For examples, in [29,30], Chen and Sun considered two kinds of SIRS epidemic model with the optimal vaccination control on heterogeneous networks. The stability conditions of equilibriums were established by using the Lyapunov function method, and the existence of optimal control and its optimal solution were studied via optimal control theory. In [31], Xu et al. have studied optimal control of a novel epidemic network with virus variation, and the existence of optimal control was also solved. Based on epidemic modeling, a network model [32] for information dissemination control was introduced, and its optimal signal distribution time to minimize the accumulated network cost was studied by using the dynamic programming. Combining the node centrality and optimal control theory, optimal control strategy of information diffusion in social networks was investigated in Kandhway and Kuri [33], which can also be used in disease spread. However, to the best of our knowledge, the existing research about optimal quarantine control on epidemic networks is seldom.

Based on the above analysis, in this paper, we will construct a new compartmental SIQS model to describe the epidemic dynamical process on complex networks, explore its stability and study its optimal quarantine control problem. The contribution of this work includes: (i) introducing a new and more reasonable epidemic model on complex networks; (ii) obtaining its stability conditions and optimal quarantine control; (iii) addressing the influence of heterogenous network structure on the optimal control.

Since the mechanism of node's birth and death plays an important role in the disease spreading, it is embedded into the constructed model to describe more realistic situations. Each node can be in one of three distinct states at each time: susceptible (S), infected (I) or quarantined (Q). In this model, we will consider two kinds of quarantine control, i.e., constant control and time-varying control. For the constant control, we mainly address the epidemic threshold and stability of equilibriums (including disease-free equilibrium and endemic equilibrium). By using Lyapunov function method and complex network theory, we obtain the epidemic threshold and stability conditions of equilibriums. The expression of epidemic threshold means that it is more important to strengthen the treatment of non-quarantined patients than quarantined patients. For the case of time-varying control, we use the optimal control theory to investigate the optimal time-varying quarantine control. The existence of optimal control is proven. Moreover, the solution of optimal control is determined.

Finally, we present some numerical simulations to verify the obtained results. In addition, based on the BA network and WS small-world network, we further explore the influence of heterogenous network structure on the optimal control. An interesting finding is that, for the BA network, there always exists an approximate $\vee$-shaped bar graph of average control strengths, while an approximate $\wedge$-shaped bar graph appears for the WS small-world network.

The remainder of this paper is organized as follows: in Section 2, we introduce an SIQS model of epidemic network with quarantine control. In Section 3, under constant control, the epidemic threshold and global stability of equilibriums are addressed. Then, in Section 4, we investigate the optimal control problem with time-varying control. In Section 5, we perform some numerical simulations. Finally, in Section 6, we conclude the paper.

2. The SIQS model of epidemic network with constant quarantine control

In our considered epidemic network, each node can be in one of three distinct states at each time: susceptible (S), infected (I) or quarantined (Q). Each susceptible node has a probability β being infected by an infected neighbor. The infected nodes are quarantined with rate $u \in [0, 1]$, which is considered as constant quarantine control in this section. Each non-quarantined infected node can recover to be susceptible with probability γ. Each quarantined infected node recovers with