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Research paper

Different characteristics and important channels between the healthy brain network and the epileptic brain network based on EEG data

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A R T I C L E I N F O

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A B S T R A C T

In this paper, firstly, based on the clinic EEG data taken from 19 channels (electrodes, nodes) on human scalp, assuming that each electrode can be represented by a two-dimensional Rulkov chaotic neuron model, and the connectivity between any two electrodes may be the non-directional coupling, the healthy brain network and the epileptic brain network are established. Secondly, different dynamical characteristics between the two networks are investigated by the master stability function analysis (MSF). The research shows that the two networks are unlikely to achieve stable synchronization when the coupling is linear and $\alpha \geq 2.8$. When the coupling is nonlinear, there exist some α such that the epileptic brain network can achieve stable synchronization for $\varepsilon \in [0.1678, 0.1694]$, while the healthy brain network is unlikely to achieve stable synchronization. Finally, based on graph theory and index of node importance, several kinds of evaluation indexes of node are calculated, such as degree, average path length and clustering coefficient. This investigation shows that the average degree and the average clustering coefficient of the epileptic brain network are larger than those of the healthy network. However, the average path length of the epileptic brain network is smaller than that of the healthy network. For other indexes, compared with the corresponding indexes of the healthy brain network, subgraph centricity, eigenvector, closeness and cumulated nomination of the epileptic brain network are increased or decreased, simultaneously. Channels *Fp*1, *T*5, *Pz* and *Fp*2 can be regarded as important focal zones for the occurrence of epilepsy.

1. Introduction

Epilepsy is a kind of neurological disorder affecting millions of the world's population. It is reported that epilepsy is mainly caused by the excessive firing and abnormal hyper-synchronization of the brain neurons [1,2]. Therefore, the reason leading to the excessive firing and the abnormal hyper-synchronization of the brain neurons has received much attention over the past decades.

On the basis of clinical data, Stamoulis *et al.* proposed two network graphs corresponding to the healthy brain and the epileptic brain in [3], respectively. They obtained part of the coupling strength among 19 channels (electrodes) and the connectivity and the directionality among 19 channels by analyzing the EEG data from five healthy adults and six patients

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with diagnosed epilepsy. What is important, they found that the healthy brain appears to be characterized by low and non-directional network coupling at rest, whereas the epileptic brain appears to be transiently and directionally synchronized.

Motivated by Stamoulis et al. [3] and Ibarz et al. [4], we build two networks corresponding to the healthy brain network and the epileptic brain network, respectively, in which the voltage of each channel (electrode) is represented by a Rulkov chaotic model, and the coupling between any two channels is assumed to be electrical and non-directional. Here, the main reason why we use a two-dimensional chaotic Rulkov neuron model to simulate a neuron is because this model can exhibit various firing activities like a biological neuron. This model can simulate different dynamic behaviors of a biological neuron such as resting, bursting, spiking and so on. So in this way, each electrode (node) can be regarded as a chaotic Rulkov model from the mathematical point of view, and each chaotic Rulkov model can mimic the activity of a neuron, and make the associated analysis and numerical simulation are relatively simple.

In view of these points, the goal of this paper is to consider the following two problems: (*i*) Do the healthy network and the epileptic network have different dynamics when the electrical coupling among neurons is linear or nonlinear? (*ii*) Can we find some channels (electrodes) that play an important role for epileptic seizures by comparing the difference of each index between the two networks?

In this paper, we at first consider the first question why the epileptic network can achieve complete synchronization while the healthy network can not achieve complete synchronization. Secondly, we manage to find some important nodes for the occurrence of epileptic seizures. One of our main results shows that the two networks are unlikely to achieve stable synchronization when the coupling is linear and $\alpha \geq 2.8$, where α is an external control parameter in Rulkov neuron model. However, when the coupling is nonlinear, there exist some α (such as $\alpha = 2.95$) such that the epileptic brain network can achieve stable synchronization when the coupling strength $\varepsilon \in [0.1678, 0.1694]$, and the healthy brain network is unlikely to achieve stable synchronization. In addition, some important channels for the epileptic network are obtained by comparison, and these important channels are $Fp1$, $Fp2$, Pz, and $T5$, respectively.

The outline of this paper is given as follows. In Section 2, we give a simple introduction of a single Rulkov map-based neuron model and its corresponding electrical coupled Rulkov neuronal model. The mathematical analysis of the master stability function is given in Section 3. In Section 4, a mathematical model which consists of 19 identical Rulkov map-based neurons is established, and the network connection patterns (regardless of the direction) are established based on the network graphs given by Stamoulis et al. [3]. Using the master stability function (MSF) method, the synchronization analysis of the two different networks are investigated in detail. In Section 5, we discuss the importance of network nodes and find important nodes in epileptic brain network.

2. Rulkov map-based neuron model and electrical coupled Rulkov neurons network model

Assumed that each channel (electrode) is simulated by a two-dimension Rulkov model. The single Rulkov map-based neuron model in [5,6] is given by:

$$\begin{aligned} x(t+1) &= f(x(t)) + y(t), \\ y(t+1) &= y(t) - \mu[x(t) - \sigma], \end{aligned} \tag{1}$$

where $x(t)$ acts as the transmembrane voltage of the neuron, $y(t)$ is the slow gating variable of the neuron, and $f(x(t)) = \frac{\alpha}{1+x^2(t)}$, t is the discrete time ($t = 0, 1, 2, \ldots$). α, σ and μ are control parameters of the signal neuron. The spiking and bursting behavior of each electrode which modeled by a single Rulkov model have been well researched in [7–9].

The network model of N identical Rulkov neurons with the electrical coupling in [10–12] is given by:

$$\begin{aligned} x_i(t+1) &= f[x_i(t)] + y_i(t) + \varepsilon \textstyle\sum_{j=1, j\neq i}^{N} g_{i,j}[h(x_j(t)) - h(x_i(t))], \\ y_i(t+1) &= y_i(t) - \mu[x_i(t) - \sigma], i = 1, 2, \ldots, N, \end{aligned} \tag{2}$$

The addition subscript i represents the ith neuron, $f(x_i(t)) = \frac{\alpha}{1+x_i^2(t)}$. Linear coupling or nonlinear coupling between nodes is determined by the form of $h(x)$. Hereinafter, each channel (neuron) can be regarded as a node in the network. υ_i is denoted as the ith neuron or the ith node of network. The parameter ε in network (2) is the electrical coupling strength. Coefficient $g_{i,j} = g_{j,i} = 1$ (or 0) if there is (or not) a non-directional connection between neurons υ_i and υ_j, but if neurons υ_i and υ_j have a directional connection and from υ_i point to υ_j, then the coefficient $g_{i,j} = 1$, $g_{j,i} = 0$.In this paper, we fix parameters at $\sigma = -1$ and $\mu = 0.001$.

3. Analysis of the master stability function

The stability and synchronization of system can be determined by the master stability function. Through the analysis of the master stability function, we can explore the stability and synchronization of the 19 channels for the healthy and the epileptic brain networks, respectively. The system (2) can be rewritten by

$$\begin{aligned} &X_i(t+1) = F(X_i(t)) + \varepsilon \textstyle\sum_{j=1, j\neq i}^{N} g_{i,j} H(h(X_j(t))), \\ &i = 1, 2, \ldots, N, \end{aligned} \tag{3}$$