Commun Nonlinear Sci Numer Simulat 65 (2018) 35–53

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Strong coexistence for a model with endogenous evolution of heterogeneous agents

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ARTICLE INFO

Article history:
Received 1 October 2017
Revised 27 February 2018
Accepted 10 May 2018
Available online 11 May 2018

Keywords:
Endogenous preferences
Multistability
Weak and strong coexistence
Bifurcation analysis

ABSTRACT

We propose two exchange economy evolutionary models with heterogeneous agents, in which the share updating mechanisms depend on the goods' consumption, described in terms of the calorie intakes. In the first setting we assume that the share updating rule is monotone in the calorie intake, while in the second framework we suppose that it is non-monotone. In both scenarios we investigate whether there may be multistability phenomena involving nontrivial market stationary equilibria at which the calorie intakes for the two groups of agents differ. The answer is negative in the first setting, while it is affirmative in the second framework.

1. Introduction

In the present paper we aim to let emerge the richness of asymptotic dynamic behaviors hidden inside the model proposed in [1] and further developed in [2]. Indeed, in order to explore the variety of asymptotic outcomes intrinsic in those works, we here relax some too restrictive assumptions made therein. More precisely, starting from the exchange economy evolutionary models with agents heterogeneous in the structure of preferences, dealt with in [1,2], in which the weights assigned to the two consumption goods in the Cobb–Douglas utility functions do not coincide across groups, we here allow also the endowments of the two goods to differ across groups of agents, in agreement with the possible extensions proposed in [1]. Like in [1,2], we assume that the growth rate of each population group is determined by a biological payoff function, which depends on the consumption of the group's agents, described in terms of the assumed calorie intake. However, we here consider two functional forms for the share updating mechanism, and we investigate how the different modeling choices influence the number of steady states and their stability. Indeed, in the first model we analyze, like in [1], the rule according to which shares are updated is monotonically increasing in the calorie intake, while in the second framework we study, as in [2], we replace the monotone population growth rate assumed in [1] with a bell-shaped map, increasing with the calorie intake up to a certain threshold value, above which it becomes decreasing.

The need to relax some of the hypotheses made in [1,2] comes from the following simple observation. In both the frameworks in [1,2], under the endowment homogeneity assumption, at the unique nontrivial (i.e., characterized by the coexistence between the two groups of agents) locally asymptotically stable equilibrium *the calorie intakes for the two groups coincide, albeit the heterogeneity in the structure of preferences*. We will refer to such feature as to weak coexistence, while

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https://doi.org/10.1016/j.cnsns.2018.05.006
1007-5704/

we will talk of strong coexistence in those cases in which there exist nontrivial locally asymptotically stable equilibria at which the calorie intakes for the two groups of agents differ. Since the weak coexistence detected in [1,2] is in deep disagreement with the empirical outcomes and limits the interpretative value of those settings, we here allow for endowment heterogeneity to try to fix the issue. In particular, in our investigation we adopt a bifurcation analysis approach.

We stress that both the share updating mechanisms we consider are empirically grounded. Namely, according to Chang and Stauber [1], a monotone population growth rate is suitable to represent the long-run centuries-old trend, as the diet of a population group affects its long-term survival. On the other hand, biological payoff functions monotonically increasing in the calorie intake well describe food regimes characterized by a calorie shortage, but they are not appropriate to represent the framework of contemporary developed countries and the negative effects of overconsumption on health and survival (see [2,3], and the references therein), which are instead properly described by bell-shaped maps. Hence, the second setting we deal with, as the one in [2], is suitable to represent the long-run centuries-old trend from the industrial revolution on, while the first framework we consider, like that in [1], is well-suited to describe the long-run centuries-old trend before the industrial revolution. We recall that evolutionary frameworks with binary choices at a collective level have been considered, for instance, in [4–7]. On the other hand, differently from those papers, the binary choice here occurs between preference structures that characterize the two groups of agents, which are embedded in a general equilibrium framework. Hence for us the agents' interaction occurs, not only in the evolutive context, but also in the market, due to the price formation mechanism. Somewhat similarly, in [8,9] the authors deal with a share updating rule embedded inside a game, i.e., inside a strategic setting, where the choices of players endowed with different dispositions depend on the choices of the other players through a best response function. In [8,9] convergence results towards regimes not characterized by the asymptotic extinction of any disposition are obtained. Hence, despite several differences, the frameworks in [8,9] share with ours the focus on the possible eventual coexistence among heterogeneous agents, as well as the evolutionary approach. Considering again [1,2] and the findings obtained therein, we recall that both settings result in one-dimensional continuous-time dynamical systems. More precisely, the analysis in those papers concerns the existence and local stability of trivial and nontrivial market stationary equilibria. In [1] at most one nontrivial market stationary equilibrium, which when exists is stable, and two trivial equilibria are found. Moreover, by construction, at the nontrivial market stationary equilibrium the calorie intakes coincide across groups. In [2], in addition to the three equilibria in [1], (up to) two additional nontrivial market stationary equilibria are detected. Furthermore, the (possibly existing) nontrivial equilibrium found in [1] may become unstable in the context considered in [2], and also the two trivial equilibria may have different dynamic behaviors with respect to [1]. In particular, unlike the framework in [1], the setting in [2] displays multistability phenomena, characterized by the presence of multiple, trivial and nontrivial, locally asymptotically stable market stationary equilibria.

We remark that, as explained in [2], multistability may be considered as a source of richness for the framework under analysis because, other parameters being equal, i.e., under the same institutional, cultural and social conditions, it allows to explain different historical trajectories and evolutionary paths. The initial conditions, leading to the various attractors, represent indeed a summary of the past history, which in the presence of multistability phenomena does matter in determining the evolution of the system. Such property, in the literature on complex systems, is also called "path-dependence" (see [10]). Moreover, in the specific context we deal with, the presence of multiple equilibria well represents the variety of historical experiences across different countries in relation to the approach they adopt towards food, diet and consequently towards obesity (consider e.g., according to Philipson and Posner [11], the different scenarios in the U.S. and in the Mediterranean countries).

On the other hand, the multistability phenomena detected in [2] are not fully satisfying from an interpretative viewpoint, as at the only involved nontrivial market stationary equilibrium the calorie intakes for both groups of agents coincide, i.e., there is just weak coexistence.

For such reason, in the present paper we add to the frameworks considered in [1,2] the heterogeneity assumption for endowments, and we study market equilibria, which link equilibrium price and optimal consumption quantities to population shares, in view of investigating whether that new heterogeneity hypothesis may generate multistability phenomena involving nontrivial market stationary equilibria with strong coexistence, at which the calorie intakes for the two groups of agents differ. As we will see, we find that the answer is negative in the setting proposed in [1], while it is affirmative in the framework in [2]. Indeed, in the former context, even under the endowment heterogeneity assumption, we do not observe multistability phenomena, since the possible outcomes are those detected in [1] and recalled above. In the framework in [2], in addition to the results obtained in that paper, we find instead multistability phenomena in which the nontrivial equilibrium is characterized by different calorie intakes for the two groups of agents. Namely, symmetric calorie intakes with respect to the desirable calorie intake, which allows maximizing the growth rate, provide the same growth rate in the framework in [2]. The possibility of observing that new kind of multistability phenomena comes from the fact that allowing for heterogeneous endowments makes the model structure less rigid and thus permits to obtain a greater richness in the outcomes, in particular, in regard to the relative position of the equilibria, and, consequently, in regard to their stability and to the chance of obtaining interesting multistability phenomena. We stress that this is a crucial difference between the homogeneous and heterogeneous endowment settings, because in case of strong coexistence the involved nontrivial equilibrium displays a deeper degree of heterogeneity between groups, not only in terms of population shares, but also from a caloric viewpoint. Finally, we perform a bifurcation analysis in order to understand how the various market stationary equilibria do emerge and disappear. We show that, according to the relative endowment values, the equilibria emerge through a