Commun Nonlinear Sci Numer Simulat 65 (2018) 70–78

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

A spatial fractional seepage model for the flow of non-Newtonian fluid in fractal porous medium

Xu Yang, Yingjie Liang*, Wen Chen*

State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Institute of Soft Matter Mechanics, College of Mechanics and Materials, Hohai University, Nanjing, Jiangsu 210098, China

ARTICLE INFO

Article history:
Received 5 February 2018
Revised 23 April 2018
Accepted 16 May 2018
Available online 17 May 2018

Keywords:
Fractional calculus
Seepage
Non-Newtonian fluid
Fractal porous medium

ABSTRACT

In the present study, a fractional seepage model (FSM) is proposed for non-Newtonian fluid via spatial fractional derivative to characterize the non-local characteristics of the non-Newtonian fluid in space and the fractal attributes of the porous medium. The analytical expressions of the permeability and the resistance are derived, in which each parameter contains clear physical meaning. The comparison between the empirical equations and our model with respect to available experimental data verifies the predictive capability of the proposed model. In addition, this study makes the first attempt to bridge the relation between the fractional derivative order and the fractal dimension of tortuosity, and may reveal the correlation between the memory of the complex fluid and characteristic pattern of the microstructure.

1. Introduction

Understanding the flow of non-Newtonian fluid in the porous medium is crucial due to its great significance in a wide range of natural and engineering fields. In contrast to Newtonian fluid, the constitutive relation of non-Newtonian fluid departs from the classical Newtonian linear correlations, i.e., shear stress is proportional to shear rate. The study on the seepage of non-Newtonian fluid serves an extensive variety of practical applications, such as flow through biological tissues [1], solute transport [2], recovery of oil and gas [3], civil engineering [4], and water circulation in the earth [5], etc. The accurate model to describe the flow of non-Newtonian fluid in the porous medium is essential to the successful design and operation of projects in related fields. It is well acknowledged that most non-Newtonian fluids exhibit history-dependent characteristics in seepage process because of the interaction between the complex fluid and tortuosity of the flow path [6,7]. However, traditional empirical equations [8–10] ignore the physics of flow as well as the sophisticated microstructures at pore level and cannot well describe the complicated phenomena [6].

One approach to address the irregular and disorder of flow path is to incorporate the fractal theory for the flow paths possessing inherent self-similarity, which is observed ubiquitous in the porous medium [11–13]. Yu et al. [14–16] proposed several fractal models for non-Newtonian fluid. However, these works focus mainly on the features of the porous medium. In fact, the tortuous character of the flow path also has an important impact on the complex fluid. Consequently, the flow pattern presents non-Boltzmann scaling [17]. A fractal Richard' equation was proposed by Sun et al. [18] to characterize the water transport in the unsaturated medium. But this method is inapplicable to non-Newtonian fluid underlying the

---

* Corresponding authors.
E-mail addresses: liangyj@hhu.edu.cn (Y. Liang), chenwen@hhu.edu.cn, paul212063@hhu.edu.cn (W. Chen).

https://doi.org/10.1016/j.cnsns.2018.05.014
1007-5704/

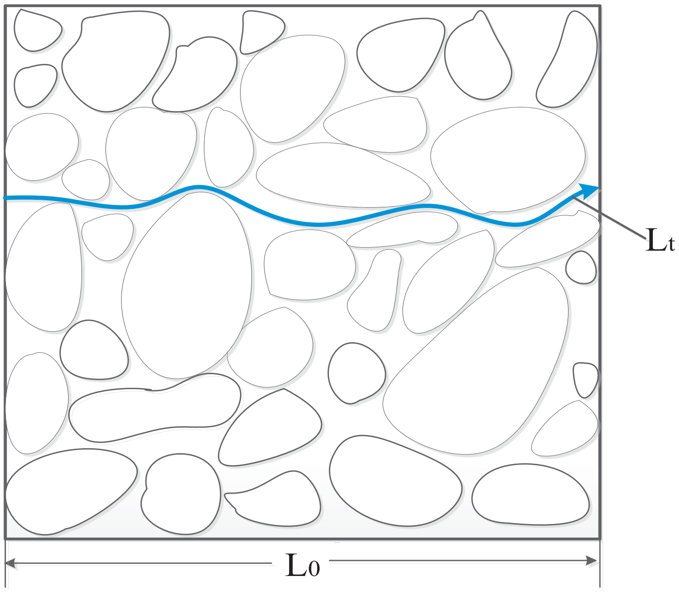

Fig. 1. The schematic diagram of tortuous flow path in porous medium.

different constitutive equation. Another approach is to use numerical simulation. However, the numerical methods for non-Newtonian fluid are usually complex and hard to implement with the serious convergence difficulties and large computation costs. Thus, a practical and efficient model with analytical expression is favored to probe the interior mechanism of seepage process.

The object of the present work is to establish a coupled model for non-Newtonian fluids via spatial fractional derivative, which combines the fractional non-Newtonian constitutive equation and the fractal attributes of the considered porous medium. The foundation and features of the proposed model will be given in details further below. The emphasis of this study lies in the investigation on permeability and resistance in the seepage process. Meanwhile, the model predictions are also verified by comparing with the Kozeny Carman equation and the Blake–Kozeny–Carman equation with respect to the available data in the literature.

2. The theory of fractal porous medium

2.1. The statistical feature of porous medium

Considerable data from field and laboratory indicate that the cumulative number (N) of pores in porous medium concerning the mescopic-structure parameters (e.g. maximum pore radius $r_{\max}$) obey the fractal scaling law [19]:

$$N(L \geq r) = \left(\frac{r_{\max}}{r}\right)^{D_f}, \tag{1}$$

where D_f is the fractal dimension of pore area, and $1 < D_f < 2$ in two topological dimension cases. The total number of pores, whose radius range from r to $r + dr$, can be obtained by differentiating Eq. (1):

$$-dN(r) = D_f r_{\max}^{D_f} r^{-(D_f+1)} dr. \tag{2}$$

Notably, $-dN(r)>0$ means that the total number of pores reduces with the increase of pore size within a given region, which coincides with the physical observation.

2.2. The flow paths of fluid in porous medium

Since the flow length by a fluid is tortuous in porous medium, which can be simplified as a bundle of tortuous capillary tube. The schematic of flow in tortuous capillary within the porous medium is illustrated in Fig. 1. Wheatcraft and Tyler [20] put forward a fractal scaling law as $L_t = \varepsilon^{1-D_T} L_0^{D_T}$, here, L_t, ε, D_T and L_0 respectively denote the tortuous length, the measuring unit, the fractal dimension of tortuosity and the straight length of the capillary. Inspired by this law, Yu [21] replaced the measuring unit ε with the diameter of capillaries $2r$ on account of the fact that $2r \ll L_0 < L_t$, then the modified fractal scaling law is formulated as

$$L_t = {L_0}^{D_T} (2r)^{1-D_T}. \tag{3}$$