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Transverse concentration distribution in an open channel flow with bed absorption: A multi-scale approach

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ABSTRACT

This study presents an analytical attempt to explore the two-dimensional concentration distribution in an open channel flow. Mei's homogenization technique is used to find the transverse concentration distribution up to third order approximations. It is well known that the Taylor dispersion model only predicts the longitudinal distribution of mean concentration and all the previous studies mostly focused on this. However, the transverse concentration distribution is important from environmental and industrial application view points. This study deduces an analytic solution of two-dimensional concentration distribution for an open channel flow with bed absorption by three-scale perturbation analysis that explores the pattern of scalar and its spreading across the channel under the effect of bed absorption. Results reveal that concentration of solute in the flowing fluid decreases with the increase in bed absorption parameter. It is also found that bed absorption causes large transverse concentration non-uniformity over the channel cross-section, especially in the downstream.

1. Introduction

Study on solute dispersion in a flowing solvent is of fundamental importance for environmental processes such as solute transport in atmosphere [1–3], river [4,5], estuaries [6–8], and wetlands [9–11]. The longitudinal dispersion of a solute in a solvent flowing through a tube of circular cross-section was first studied by Taylor [12]. In this pioneering work, he showed that transverse mean concentration of a solute was not only simply advected by mean velocity and diffused by molecular diffusivity, instead, the velocity shear across the tube increased the diffusion rate in the flow direction. This enhanced diffusion was termed as Taylor dispersion. This predominant feature has attracted intensive studies and founded extensive applications [13–18].

Taylor dispersion is basically an idea for the evaluation of transverse mean concentration, which is leading to a Gaussian distribution. The main feature of such a distribution is to predict the peak or the highest concentration value of the evolution curve, which has essential implications for environmental risk assessment [19]. Taylor showed that for large times, transverse mean concentration satisfies a one-dimensional diffusion like equation. In order to study the solute dispersion processes, several analytical methods are developed over time that include Aris' method of moments [20,21], Gill's method of mean concentration expansion [22–24], homogenization technique [25–27], delay-diffusion equation [28]. In addition, there are also some numerical attempts [29–32], which provide the evolution of the mean concentration cloud for the en-

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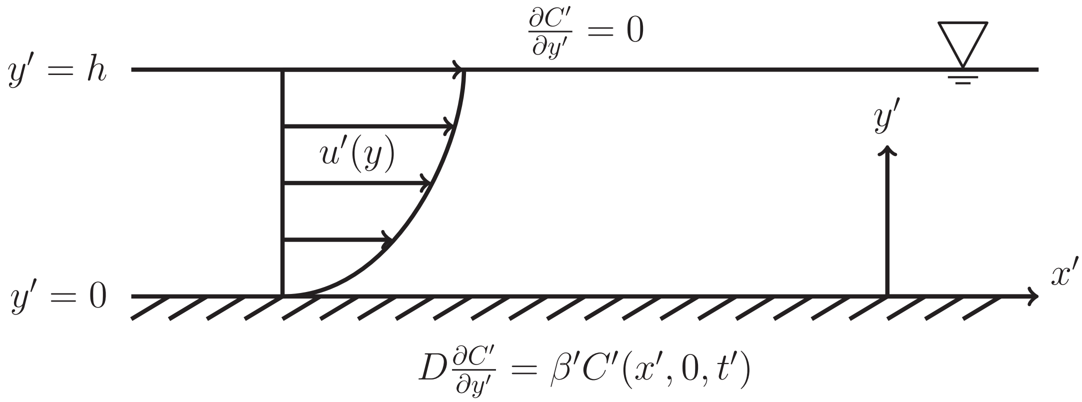

Fig. 1. Sketch for the instantaneous release of solute in open channel flow with bed absorption.

tire process. In this process, one important phenomenon is that the mean concentration approaches the normal distribution at the initial stage. Chatwin [33] proposed an asymptotic series expansion of concentration and studied how the mean concentration approached to longitudinal normality. He proposed a time scale 1.0 r^2/D (r is the tube radius and D is the molecular diffusivity), which was an estimate of time scale for the mean concentration to reach longitudinal normality.

Literature on solute transport, until last decade, mostly dealt with the mean concentration distribution [34–39] rather than transverse concentration distribution. In many environment and industrial processes, transverse concentration distribution is more useful compared to the longitudinal mean concentration. Wu and Chen [40,41] systemically analyzed the transverse uniformity of concentration cloud for laminar steady flows. They proposed a time scale 10 r^2/D, which was an estimate of time scale for the real concentration to reach transverse uniformity.

Taylor dispersion process in an open channel flow with boundary reaction has been studied extensively by many. However, efforts have been limited to determine the Taylor dispersion coefficients in the one-dimensional effective diffusion equation for the mean concentration [42,43]. The exact peak position of real concentration distribution is a concern, rather than that of the mean concentration distribution. Wang and Chen [44] explored the transverse concentration distribution in an open channel flow with bed absorption by extending the Aris–Gill expansion method and analyzed the initial skewed transient behaviors of the solute cloud. Sometimes, for a long channel, in order to know the solute concentration at locations far from the solute source, one does not need to know the initial transient behavior of the solute cloud.

In this paper, an analytical study is presented to explore two-dimensional concentration distribution in an open channel flow with absorbing channel bed. Simple analytical expressions are derived for transverse concentration distribution with the help of multi-scale method of homogenization proposed by Mei et al. [45]. The method of multi-scale comprises of techniques used to construct effective macro-scale models from the complicated micro-scale models. The main idea here is to obtain effective equations for the slow time scale variable over long time scales by averaging over rapid variations of the fast variables. The need of this technique lies with the fact that the traditional one scale approaches are not accurate enough at macro level and not efficient enough at micro level. This study explores mean and real concentration distributions after an initial time when transient behavior completely dies out. This study also concerns about the non-uniformity of concentration variation over the channel cross-section and effects of bed absorption on it.

The specific objectives of this work are: (I) to present a multi-scale analysis for the concentration distribution in an open channel flow with bed absorption, (II) to obtain analytical expressions for the transverse real concentrations up to third order approximations, (III) to observe the effects of bed absorption on transverse variation of concentration distribution, (IV) to discuss the uniformity in transverse real concentration over the channel cross-section.

2. Formulation of the problem

A laminar, fully developed, unidirectional open channel flow with a separation width h is considered. A Cartesian coordinate system has been used assuming longitudinal direction as x'-axis and vertical direction as y'-axis. The solute is considered to undergo an irreversible bed absorption at the channel bed and to disperse throughout the channel under a flow velocity $u'(y')$, which is given as

$$u'(y') = u_0\left(2\frac{y'}{h} - \frac{y'^2}{h^2}\right), \tag{1}$$

where u_0 is the flow velocity of the free surface. Schematic diagram of flow geometry is given in Fig. 1.

The problem for the transport of the solute can be formulated as follows:

$$\frac{\partial C'}{\partial t'} + u'\frac{\partial C'}{\partial x'} = D\frac{\partial^2 C'}{\partial x'^2} + D\frac{\partial^2 C'}{\partial y'^2}, \quad 0 < y' < h, \tag{2}$$