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Research paper

General high–order breathers and rogue waves in the $(3+1)$-dimensional KP–Boussinesq equation

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ABSTRACT

In this work, we investigate the $(3+1)$-dimensional KP–Boussinesq equation, which can be used to describe the nonlinear dynamic behavior in scientific and engineering applications. We derive general high–order soliton solutions by using the Hirota's bilinear method combined with the perturbation expansion technique. We also obtain periodic solutions comprising of high-order breathers, periodic line waves, and mixed solutions consisting of breathers and periodic line waves upon selecting particular parameter constraints of the obtained soliton solutions. Furthermore, smooth rational solutions are generated by taking a long wave limit of the soliton solutions. These smooth rational solutions include high-order rogue waves, high–order lumps, and hybrid solutions consisting of lumps and line rogue waves. To better understand the dynamical behaviors of these solutions, we discuss some illustrative graphical analyses. It is expected that our results can enrich the dynamical behavior of the $(3+1)$-dimensional nonlinear evolution equations of other forms.

1. Introduction

During last decades, nonlinear evolution equations (NLEEs) are well used to model a wide variety of nonlinear phenomena in many scientific fields such as plasma physics, nonlinear optics, fluid dynamics, solid state physics, electromagnetic waves, and many others. The significant feature of nonlinear evolution equations have attracted an increasingly research works from mathematicians, physicists, and engineers. The research of nonlinear physics phenomena is flourishing because of the rich findings of these equations. The determination of exact soliton solutions to nonlinear wave equations is of great value to understand widely different physical phenomena.

As stated earlier, interests have increased to investigate the nonlinear evolution equations particularly the completely integrable NLEEs, which display significant properties as the soliton solutions, infinite number of conservation laws, symmetries and Hamiltonian structures [1,2]. Due to fact that solutions of NLEEs can provide much physical information and more insight into the physical aspects and then lead to further applications, deriving solutions to nonlinear problems plays a significance role in nonlinear science. Indeed, in order to determine the solutions to NLEEs and to examine the physical

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properties of these solutions, there are many powerful methods which have been used, such as Darboux transformation [3,4], Hirota bilinear method [5,6], inverse scattering transform method [1], the homogeneous balance method [7,8], the Lie group method [9,10], the direct method [11–18] and so on. The search, by using such a variety of schemes, is always for a closed form analytical solution that may lead to new developments. Several kinds of solutions are usually obtained, such as soliton, peakons, cuspons, kinks, positons, breather, lump, rogue wave solutions and many others.

Rogue waves, also called freak waves, giant waves, great waves, ghost waves, killer waves, etc, are unexpectedly large displacements evolving from an otherwise calm sea state background, which were originally observed in the oceans [19–22]. Recently, more works were devoted to study such rare extreme events in areas as diverse as nonlinear optics [23–25], Bose–Einstein condensates [26,27], plasma physics [28,29], and so on. A possible mechanism for the formation of rogue waves is associated with modulation instability [30–33]. Mathematically, fundamental first-order rogue wave solution was first reported by Peregrine [34], which is localized in both space and time. The amplitude of the first-order rogue wave reaches three times of the height of background, and then finally decays algebraically to the background. Higher-order rogue waves in the Nonlinear Schrödinger Equations (NLS) were reported in many articles [35–41] and in some of the references therein. Recently, a variety of nonlinear soliton equations have been verified possessing rogue wave solutions [42–56]. Two recent useful works [57,58] have provided a remarkable review on the rogue waves from the physical view.

In this work, we aim to investigate the $(3+1)$-dimensional generalized KP–Boussinesq equation, given as

$$u_{xxxy} + 3(u_x u_y)_x + u_{ty} + u_{tx} + u_{tt} - u_{zz} = 0, \tag{1}$$

where $u = u(x, y, z, t)$ is a differentiable function. This equation was introduced by Wazwaz and El–Tantawy [59]. As the Boussinesq equation, the KP–Boussinesq (1) is also a nonlinear PDE of second order in time t, which models both right and left-going waves. The KP–Boussinesq (1) is unlike the integrable KdV and the KP equations which are given by first-order PDEs in time. The single- and double-soliton solutions were studied by Wazwaz and El–Tantawy [59]. However, to the best of authors' knowledge, higher-order solitons, breathers and rogue waves for the KP–Boussinesq (1) have not been investigated before.

Motivated by the goal to make further progress on the KP–Boussinesq (1), we aim to construct general higher-order soliton, breather and rogue wave solutions, and to explore their fascinating dynamical behaviors. The rest of the paper is organized as follows: In Section 2, general higher-order solitons, breathers and rogue wave solutions are obtained, and dynamics of these solutions are discussed in detail. Our results are summarised in Section 3.

2. Solutions of the $(3+1)$-dimensional KP–Boussinesq equation

In this section, we focus on families of solutions to the $(3+1)$-dimensional KP–Boussinesq Eq. (1). Under the variable transformation

$$u = 2(\log f)_x, \tag{2}$$

then the bilinear form of the $(3+1)$-dimensional KP–Boussinesq Eq. (1) is generated as

$$(D_x^3 D_y + D_t D_x + D_t D_y + D_t^2 - D_z^2) f \cdot f = 0. \tag{3}$$

Here f is a real function of variables x, y, z, t, and the operator D is the Hirota's bilinear differential operator [5] defined by

$$H(D_x, D_y, D_t,)F(x, y, t \cdots) \cdot G(x, y, t, \cdots) = H(\partial_x - \partial_{x'}, \partial_y - \partial_{y'}, \partial_t - \partial_{t'}, \cdots)F(x, y, t, \cdots)G(x', y', t', \cdots)|_{x'=x, y'=y, t'=t},$$

where H is a polynomial of D_x, D_y, D_t, $\cdots$.

Below, we construct soliton solutions to the $(3+1)$-dimensional generalized KP–Boussinesq Eq. (1) by using the Hirota's bilinear method combined with the perturbation expansion [5]. By selecting particular complex conjugation of the obtained soliton solutions, a family of analytical solutions, termed breathers, can be systematically derived. Further, by taking a long wave limit, rational solutions consisting of lumps and line rogue waves, and semi-rational solutions consisting of periodic line waves, rogue waves, lumps, and solitons, can be generated.

2.1. First-order breather and rational solutions

We first start from the two-soliton solution. To this end, we take f in (2) being the following formal form:

$$f = 1 + \epsilon f_1 + \epsilon^2 f_2, \tag{4}$$

with

$$f_1 = e^{\eta_1} + e^{\eta_2}, \quad f_2 = e^{\eta_1 + \eta_2 + A_{12}}, \tag{5}$$

where

$$\eta_s = p_s x + k_s y + \omega_s t + l_s z + \phi_s, s = 1, 2, \tag{6}$$