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Research paper

Experimental evidence of power law reinjection in chaotic intermittency

Ezequiel del Rio[a,*], Sergio Elaskar[a,b]

[a] Dpto. Física Aplicada. ETSI Aeronáutica y del Espacio. Universidad Politécnica de Madrid, Plaza Cardenal Cisneros 3. 28040 Madrid, Spain

[b] Departamento de Aeronáutica, FCEFyN Universidad Nacional de Córdoba, Instituto de Estudios Avanzados en Ingeniería y Tecnología CONICET and Universidad Nacional de Córdoba, Argentina

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ABSTRACT

The main classical results on classical chaotic intermittency were based on a uniform reinjection on the laminar region. Recently, for a wide class of 1D maps, a generalized power law reinjection was found, so the classical theory of intermittency was generalized for 1D-maps. We propose experimental evidences that this generalization works also in an experiment based on an analog circuit. In the noisy case, we also found good agreement with a recently proposed theory for this scenario.

1. Introduction

Intermittency is a particular route to the deterministic chaos characterized by spontaneous transitions between laminar and chaotic dynamics. For the first time this concept has been introduced by Pomeau and Maneville in the context of the Lorenz system [1,2]. Later intermittency has been found in a variety of different systems including, for example, periodically forced nonlinear oscillators, Rayleigh-Bénard convection, derivative nonlinear Schrödinger equation, and in development of turbulence in hydrodynamics (see e.g. Refs. [3,4]). Proper qualitative and quantitative characterizations of intermittency based on experimental data are especially useful for studying problems with partial or complete lack of knowledge on exact governing equations, as it frequently happens e.g. in Economics, Biology, and Medicine (see e.g. Refs. [5,6]). In this case special attention has to be payed to the length of data sets required for robust estimation of the model parameters.

According to Pomeau and Manneville, intermittency can be classified into three types depending on the local geometry of their Poincaré map: type-I for quadratic maps and type-II and type-III for cubic ones [7]. However, other non linear types had been reported [8–10], hence the local laminar dynamics of type-I intermittency is determined by the 1D map in the form :

$$x_{n+1} = \varepsilon + x_n + a\, x_n^p \qquad \text{Type-I} \tag{1}$$

* Corresponding author.
E-mail address: ezequiel.delrio@upm.es (E.d. Rio).

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where $a > 0$ accounts for the weight of the nonlinear component determined by the exponent p. The laminar behavior of type-II and type-III are described by the maps:

$$x_{n+1} = (1+\varepsilon)x_n + ax_n^p \qquad \text{Type-II} \tag{2}$$

$$x_{n+1} = -(1+\varepsilon)x_n - ax_n^p \qquad \text{Type-III} \tag{3}$$

In all the types, the fixed point of the system becomes unstable for positive values of the parameter ε. Traditionally was used $p = 2$ for type I intermittency and $p = 3$ for type II and III. However, those restrictions are actually not necessary [11]. Another condition for a one-dimensional map to possess intermittency is to have a reinjection mechanism mapping back the trajectories from the chaotic zone into the laminar one. This mechanism is described by the so-called reinjection probability density (RPD), which is determined by the chaotic dynamics of the system itself. The function RPD together with the local map determine all the dynamical properties of the system. In general it is difficult to get an analytical expression for the RPD, hence different approximation have been used. The most common approximation used to obtain many classical results on intermittency theory have been to consider the RPD uniform. and thus independent of the reinjection point [3,7,12–17], however, works fine in a few model cases only [12,14,16]. Another approach deals with the other limit, δ-function like RPD. It considers reinjection into a given point in the presence of noise [15,18].

Recently to describe the reinjection mechanism of a wide class 1D maps exhibiting intermittency it was introduced a generalized RPD, a parametric power law function depending on a free parameter $m \in (0, 1)$. The generalized RPD includes the uniform reinjection as a particular case $m = 1/2$ [9,19]. We showed that the shape of the generalized RPD is determined by the behavior of trajectories within chaotic regime in a vicinity of a point in the Poincare map with infinite or zero tangent [20].

The shape of the new RPD enables analytic estimation the fundamental characteristic of the intermittency, that is, the probability density of the length of laminar phase $\psi(l)$ where l approximates the number of iterations in the laminar region, i.e. the length of the laminar phase. Note that, as the function $\psi(l)$ can be estimated for time series, it is usually used to identify the intermittency type by comparing with the analytic estimation.

The new RPD has been found in many 1D map and the analytical predictions for RPD and for$\psi(l)$ have been numerically confirmed even in the pathological cases of intermittency described in the literature. This case are known by their signiant deviation of the main characteristics (e.g. the length of the laminar phase) from those predicted by the classical theory. It have been shown that the generalized reinjection probability density provides faithful description of anomalous and standard intermittencies in the unified framework [20,22].

Because the noise is always present in experiments, it is of a fundamental importance to know the effect of noise on the intermittency phenomenon, in particular on the RPD. There are many papers devoted to study noise effect on chaotic intermittency, by means of the normalization group analysis [8], or by using the Fokker–Plank equation [17,18,26,27] among the others. Note that, in spite of the fact that the noise affects the whole region where the system dynamics takes place, classically, the researches are devoted to the noise effect on the laminar region of the Poincaré map and there was no study in classical theory of intermittency focused on the effect of noise on the RPD.

Actually, the RPD described by a power law introduces a novel scenario because, whereas the classical uniform reinjection should remain constant under a wide class of noise distributions, the new RPD can be affected by the noise. For 1D-map, recently an analytical approach to the noise reinjection probability density (NRPD) has been present [23–25,28]. Note that the probability density of the laminar lengths $\psi(l)$ depend on the reinjection probability density, hence the NRPD also determines a noisy $\psi(l)$. For a review of the generalized theory see [11].

The theory of intermittency sketched before has been developed for 1D map. However, continuous systems that contract volume in phase spaces can be described by the 1D maps [29], hence, it is possible, at least theoretically, to extend the new intermittency framework to continuous systems. In spite of that, there is has not been reported experimental confirmation in continuous systems.

This demands further investigations. In this paper we apply the previously described the intermittency theory to experimental data coming from an analog circuit. We provide an experimental confirmation of the relevant results of the new intermittency theory. In particular, The RPD base on a power law found in a wide class of 1D map also describe accurately the reinjection found experimentally in our continuous system. Moreover, the aims of this paper is also to show that the RPD observed in experimental continuous dynamical systems is affected by the noise of the system as it has recently been analytically and numerically reported for 1D-maps.

The rest of the paper is organized as follows. The Section 2 is devoted to make a brief introduction of the intermittency theory that we need to evaluate the experimental result. In Section 3 we present the model and the experimental setup. Section 4 deals with the experimental Poincaré map. Results on the experimental noiseless power law RPD are present in Section 5 whereas Section 6 is devoted to the noisy case. The mains conclusions are present in Section 7.

2. Mathematical background

Before embarking in the experiment description, let us summarize the concept that we will used to analyze the experimental results. In absence of noise, the generalized RPD is given by the next power law [9,10,19].

$$\phi(x) = b(x - \hat{x})^{\alpha} \tag{4}$$