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Research paper

A nonlinear dynamic age-structured model of e-commerce in spain: Stability analysis of the equilibrium by delay and stochastic perturbations

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ABSTRACT

First, we propose a deterministic age-structured epidemiological model to study the diffusion of e-commerce in Spain. Afterwards, we determine the parameters (death, birth and growth rates) of the underlying demographic model as well as the parameters (transmission of the use of e-commerce rates) of the proposed epidemiological model that best fit real data retrieved from the Spanish National Statistical Institute. Motivated by the two following facts: first the dynamics of acquiring the use of a new technology as e-commerce is mainly driven by the feedback after interacting with our peers (family, friends, mates, mass media, etc.), hence having a certain delay, and second the inherent uncertainty of sampled real data and the social complexity of the phenomena under analysis, we introduce aftereffect and stochastic perturbations in the initial deterministic model. This leads to a delayed stochastic model for e-commerce. We then investigate sufficient conditions in order to guarantee the stability in probability of the equilibrium point of the dynamic e-commerce delayed stochastic model. Our theoretical findings are numerically illustrated using real data.

1. Introduction

Electronic commerce (in short e-commerce) is the use of advanced electronic technology for a wide range of on-line business activities for goods and services. E-commerce is gradually extending to the economic mainstream and business core aspects. E-commerce has provided a new way of doing business all over the world using the Internet. Modelling the diffusion of e-commerce is extremely important for business investors and policymakers for effective planning and better understanding the dynamics of this complex transactional process. A number of mathematical models have been proposed to study e-commerce using different approaches. Here, we highlight contributions based upon Making-Decision Theory mainly oriented towards the design of recommender systems [1,2] and the measure of quality quality of services and business [3,4]. These contributions rely on operational research (decision making support systems, multi-criteria optimization, etc.) and statistical techniques (Bayesian analysis, Petri nets, etc.). Pioneering contributions dealing with the social diffusion of new technologies using mathematical models based on differential equations include [5–7]. From the point of view of dynamical

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systems, the study of e-commerce has been analysed in a few contributions. In [8] the authors present a competition model of e-commerce sites and they perform a planar qualitative analysis. Afterwards, Li and Siming explored the effects of competition in e-commerce web sites via mathematical models based on ordinary differential equations, [9,10]. These interesting studies include a qualitative equilibrium analysis and numerical simulations of the competition dynamics. In [11] some of the authors of the present paper proposed an age-structured compartmental mathematical model (similar to the ones used to model epidemics [12]) to describe the dynamics of e-commerce using real data from the Spanish National Statistical Institute (INE). This study is performed by combining two mathematical models, the first one is a demographic model providing certain demographic parameters required in the formulation of the second model, which is addressed to describe the diffusion of e-commerce. According to the available data from the Spanish INE, population was divided into six cohorts. The results obtained in [11] are quite good despite predictions were performed on the horizon 2010–2012 by fitting sampled data corresponding to only three available years at that time (2006–2008). The inclusion of the age-structured model is a difficult issue that we are going to consider in this paper. Some interesting contributions where the age structure has been considered in the context of mathematical modeling, can be seen, for example, in [13–16].

We are aware that significant features of e-commerce are not contained in the formulation of the mathematical model proposed in [11]. On the one hand, according to [17], our habits are influenced by the habits of the people in our social network. This can be also applied to the habit of the use of e-commerce that can be transmitted by peer pressure or social contact among family, friends, mates, etc. However, the adoption of this technology does not take place immediately after such encounters, but it requires a certain time lag (delay). On the other hand, the success of *contagion* depends on a number of complex human and business factors whose nature is random (social contacts, purchase behavior, personality, confidence, impulsiveness, technology integration, etc., [18,19]). Furthermore, real data required to fit the proposed model contains sampling errors and hence uncertainty. These reasons aim us to propose an epidemiological model to describe the dynamics of the use of e-commerce in Spain that considers in its formulation both delay and randomness. There are two main approaches to deal with delay: first, random fractional differential equations [20] and second, random delay differential equations [21]. In this paper, we follow the latter approach. As it has been reported in previous contributions [11], the use of technologies, and in particular the e-commerce, is strongly related to the age of users. This key feature must be taking into account in the mathematical formulation as we did in our previous contribution [11]. At this point is important to point out that we have made the decision of aggregating data from Spanish INE into two subpopulation, Group 1: persons aged $15-44$ years old (y.o.) and Group 2: persons aged $45-74$ y.o. Apart from the feasibility of the subsequent mathematical treatment of the mathematical model, this decision has been made in agreement with the significant differences of the use of e-commerce between these two age groups found in data collected from Spanish INE [22,23]. Furthermore, it can be checked from this statistical source that the percentage of people younger than 14 y.o. and older than 74 y.o. buying by the Internet is practically negligible. Thus, in this paper we propose a mathematical model for studying the dynamics of e-commerce that combines the aforementioned bare-bones factors: an underlying age-structured demographical model, peer-pressure (contagion) to account for the diffusion of this technology, delay and randomness effects.

As we will see later, we consider uncertainty via stochastic perturbations from the equilibrium point since our model must be able to capture eventual changes that may happen about the steady point because of social and business factors affecting the dynamics of the e-commerce. This is a key issue in our subsequent analysis both from a practical and theoretical standpoints. Indeed, if the mathematical model is reliable, it is expected the numerical results in real-world (using real data of Spanish e-commerce) remain stable except perhaps in the case of large perturbations while the stability analysis has an intrinsic mathematical interest. Both questions lead to investigate the maximum size of stochastic perturbations in order to guarantee the stochastic stability of the equilibrium point.

More specifically, we will assume that the dynamics of e-commerce model with delay is exposed to additive stochastic perturbations of White Noise-type that are directly proportional to the deviation of the current state of the system from the steady state or equilibrium point. From a mathematical modelling standpoint and, on the basis of the Limit Central Theorem, it must be pointed out that the large number of independent random factors, previously mentioned, that may affect the dynamics of e-commerce diffusion supports the consideration of White Noise process which is a Gaussian stationary process with constant spectral density [24,25]. Such type of stochastic perturbations first was proposed in [26,27]. One of the key points of this hypothesis is that the equilibrium point is the solution of the stochastic system too. In this case, the influence of the stochastic perturbations on the considered system is small enough in the neighborhood of the equilibrium point and big enough if the system state is far enough from the equilibrium point.

The considered nonlinear system is then linearized in the neighborhood of the positive equilibrium point, and sufficient condition for asymptotic mean square stability of the zero solution of the constructed linear system is obtained via the Kolmanovskii–Shaikhet general method of Lyapunov functionals construction (GMLFC), that is used for stability investigation of stochastic functional-differential and difference equations [21,28–30]. This way of stability investigation was successfully used in different mathematical models formulated via systems with delays: SIR epidemic model [26], predator-prey model [27,31], social epidemic models [32,33] and Nicholson blowflies model [34], for example.

On the basis of the aforementioned approach, the main objective of this paper is twofold. First, from an applied standpoint, to propose a mathematical model able to describe the diffusion of e-commerce in Spain using real data and, second, from a mathematical point of view, to perform a stability analysis of the equilibrium by delay and stochastic perturbations. As a consequence, the new model can be regarded, in some aspects, as an extension of the one presented in [11] since in its formulation it includes delay and randomness, but reducing the number of subpopulation of the underlying demographic