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Research paper

Comb model for the anomalous diffusion with dual-phase-lag constitutive relation

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ABSTRACT

As a development of the Fick's model, the dual-phase-lag constitutive relationship with macroscopic and microscopic relaxation characteristics is introduced to describe the anomalous diffusion in comb model. The Dirac delta function in the formulated governing equation represents the special spatial structure of comb model that the horizontal current only exists on the x axis. Solutions are obtained by analytical method with Laplace transform and Fourier transform. The dependence of concentration field and mean square displacement on different parameters are presented and discussed. Results show that the macroscopic and microscopic relaxation parameters have opposite effects on the particle distribution and mean square displacement. Furthermore, four significant results with constant 1/2 are concluded, namely the product of the particle number and the mean square displacement on the x axis equals to 1/2, the exponent of mean square displacement is 1/2 at the special case $\tau_q = \tau_P$, an asymptotic form of mean square displacement (MSD$\sim t^{1/2}$ as $t \to 0, \infty$) is obtained as well at the short time behavior and the long time behavior.

1. Introduction

Anomalous diffusion is a non-Markov and non-local movement with a fractional relationship $\langle x^2(t)\rangle \sim t^\gamma$ [1], where $\gamma > 1$ refers to the anomalous superdiffusion, $\gamma < 1$ denotes the anomalous subdiffusion while $\gamma = 1$ corresponds to the normal diffusion. The diffusion in comb model is a special case of anomalous diffusion. As Fig. 1 shows, the comb model consists of a one-dimensional backbone with fingers of infinite length. The special behavior [2–3] is that the displacement in the x direction is possible only along the x axis with diffusion coefficient $D_1\delta(y)$ while the diffusion along the y direction plays a role of traps with traditional diffusion coefficient D_0, here $\delta(y)$ refers to the Dirac delta function, D_1 and D_0 are nonnegative constant. It has attracted considerable attention due to its simplicity and ability to reproduce subdiffusive behaviors of disordered systems [4–5], such as the fractional transport of cancer cells due to self-entrapping [6], the reaction-subdiffusion front propagation in spiny dendrites [7], and the reaction front propagation of actin polymerization [8].

As the basis to study the diffusion process, the constitutive model refers to the relationship between the diffusion flux and concentration gradient. The classical constitutive relation [9–10] to describe the anomalous diffusion in comb model is

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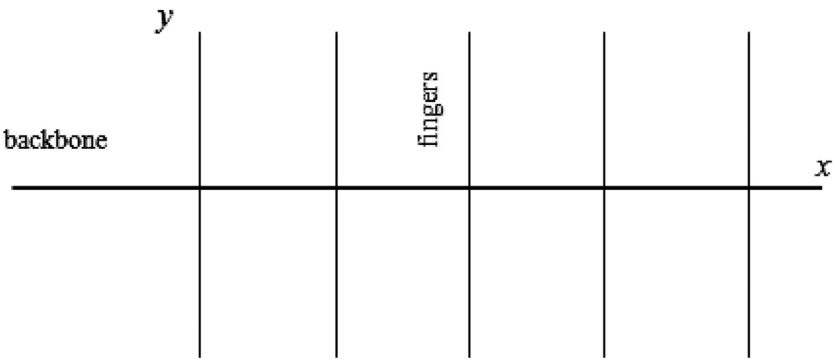

Fig. 1. Schematic drawing of the comb model.

based on the Fick's law with a linear form, given as:

$$\vec{J} = \left(-D_1\delta(y)\frac{\partial P}{\partial x}, -D_0\frac{\partial P}{\partial y}\right), \tag{1}$$

where the symbol $\vec{J}$ is the diffusion flux vector, $P = P(x, y, t)$ refers to the distribution function at the special positions (x, y) and time t. Here, x denotes the direction along the backbone of the comb structure while y measures the distance along the fingers away from the backbone.

As is well known, the classical Fick's model contains paradox with infinite propagation velocity [11–12]. In order to overcome this paradox, the macroscopic relaxation parameter is introduced by Cattaneo [13] to modify the classical model. As a development of the Fick's model, the Cattaneo model has widespread applications in practical problem, such as the reaction-diffusion problem in a disordered porous medium [14], the non-isothermal incompressible fluid flow involving heat transfer and friction [15], the laser short-pulse heating of a solid surface [16].

However, the Cattaneo model only takes account the fast-transient effects. For a complete diffusion process, it should consider not only the macroscopic description but also the interactions between the microscopic structures [17]. As a generalization of the Cattaneo model, the dual-phase-lag model is proposed by Tzou [18–19] which contains both the macroscopic relaxation parameter and the microscopic one. It describes the actual problem more accurately and attracts many scholars to study. Among the researchers, Liu [20] analyzed the dual-phase-lag thermal behavior in two-layered thin films with an interface thermal resistance, concluding that the lagging thermal behavior depends on the magnitude of the macroscopic and microscopic relaxation parameters. Using the dual-phase-lag model, Ghazanfarian and Abbassi [21] simulated micro- and nano-scale heat conduction within a thin slab for Knudsen numbers more than 0.1 including phonon scattering boundary condition by the numerical and analytical methods. Abouelregal [22] investigated rayleigh waves in a thermoelastic solid half space using dual-phase-lag model by numerical method and the effect of the coupling parameter and phase-lags was showed graphically. More references related to the studies of the dual-phase-lag model can be seen in Refs. [23–25].

In actual, the study on the anomalous diffusion in comb model with modified constitutive equations is a promising direction and a lot of modified constitution relationships have been analyzed and discussed. Iomin [26] proposed the time fractional diffusion flux along the fingers to describe the anomalous diffusion in comb model. Tateishi [27–28] analyzed the anomalous diffusion in comb model by changing the integer derivative operator in Fick's model as the fractional one. Liu et al. [29–31] analyzed the anomalous diffusion in comb model by adopting the fractional Cattaneo model and the fractional Cattaneo–Christov model. Méndez et al. [32–33] displayed a variety of macroscopic transport regimes and formulated the mesoscopic description of the random walk on the comb.

Inspired by the studies mentioned above, in this paper, the dual-phase-lag model containing the macroscopic and microscopic relaxation parameters is introduced to study the anomalous diffusion in comb model. The governing equation with the highest order of two is solved by analytical method where the Laplace and Fourier transforms are applied. The product of the particle number and the mean square displacement on the x axis is computed. Besides, three special cases about the mean square displacement, such as the case of the equal macroscopic and microscopic relaxation parameters, the limit cases as time tends to zero and infinite are analyzed. In order to verify the correctness of analytical solution, the comparisons with the degenerated solutions are presented. The particle distribution and the mean square displacement with the influences of different parameters are discussed and the main results and conclusions are also analyzed.