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Research paper

Large stable oscillations due to Hopf bifurcations in amplitude dynamics of colliding soliton sequences

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ABSTRACT

We demonstrate that the amplitudes of optical solitons in nonlinear multisequence optical waveguide coupler systems with weak linear and cubic gain-loss exhibit large stable oscillations along ultra-long distances. The large stable oscillations are caused by supercritical Hopf bifurcations of the equilibrium states of the Lotka–Volterra (LV) models for dynamics of soliton amplitudes. The predictions of the LV models are confirmed by numerical simulations with the coupled cubic nonlinear Schrödinger (NLS) propagation models with $2 \le N \le 4$ pulse sequences. Thus, we provide the first demonstration of intermediate nonlinear amplitude dynamics in multisequence soliton systems, described by the cubic NLS equation. Our findings are also an important step towards realization of spatio-temporal chaos with multiple periodic sequences of colliding NLS solitons.

1. Introduction

The cubic nonlinear Schrödinger (NLS) equation, which describes wave propagation in the presence of second-order dispersion and cubic (Kerr) nonlinearity, is one of the most widely used nonlinear wave models in physics. It was successfully employed to describe water wave dynamics [1,2], Bose–Einstein condensates [3,4], and pulse propagation in nonlinear optical waveguides [5,6]. The fundamental NLS solitons are the most ubiquitous solutions of the cubic NLS equation due to their stability. Indeed, stable dynamics of single NLS solitons and of a single periodic sequence of NLS solitons has been observed in a wide range of physical systems [1–6]. However, the situation is very different for propagation of multiple periodic soliton sequences. Such multisequence propagation setups are of particular interest in nonlinear broadband (multichannel) optical waveguide systems [5–7]. In these waveguide systems, the solitons in each periodic sequence propagate with the same frequency and group velocity, but the frequency and group velocity are different for solitons from different sequences [5–7]. As a result, intersequence soliton collisions are frequent and can lead to significant amplitude and frequency shifts and to severe transmission degradation. In fact, multichannel transmission with NLS solitons is unstable due to resonant emission of small-amplitude waves [7–9]. Considering the ubiquity of the fundamental NLS soliton and of the single soliton sequence, the fact that stable long-distance propagation of multiple sequences of fundamental solitons has

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not been demonstrated in any system described by the cubic NLS equation is quite troubling. In particular, one would not expect intermediate or strongly nonlinear dynamics of soliton amplitudes in these systems.

In Refs. [9–14], we developed a general method for stabilizing dynamics of soliton amplitudes in nonlinear multisequence optical waveguide systems with nonlinear dissipation. The method is based on showing that amplitude dynamics induced by nonlinear dissipation in N-sequence waveguide systems can be approximately described by N-dimensional Lotka–Volterra (LV) models. Stability analysis of the equilibrium states of the LV models can then be used for realizing stable amplitude dynamics along ultra-long distances. However, due to the inherent instability of multichannel soliton-based transmission against radiation emission, the distances along which stable amplitude dynamics was observed in numerical simulations were initially limited to a few hundred dispersion lengths [11,12]. Significant increase in the stable propagation distances was enabled by the introduction of frequency dependent linear gain-loss in N-waveguide couplers [8,9]. The limiting cause for transmission instability in the latter systems was associated with radiation emission due to the effects of dissipative perturbations on single-soliton propagation [9]. Therefore, this process is a serious obstacle for observing intermediate and strongly nonlinear amplitude dynamics in multichannel transmission with NLS solitons. Indeed, in all previous studies of multichannel soliton-based transmission, the dissipation-induced amplitude dynamics was only weakly nonlinear [9–14]. Furthermore, intermediate or strongly nonlinear amplitude dynamics has not yet been demonstrated in any multisequence soliton system, described by the cubic NLS equation.

A common mechanism for inducing intermediate nonlinear dynamics is by means of supercritical Hopf bifurcations [15–18]. In this case, as the value of a physical parameter is changed beyond some threshold value, a stable equilibrium state of the dynamical model becomes unstable, and a stable limit cycle about the unstable equilibrium state appears [15,16]. As a result, for parameter values larger than the threshold value, the system exhibits stable oscillations with relatively large amplitudes, i.e., intermediate nonlinear amplitude dynamics. Supercritical Hopf bifurcations occur in many physical systems, including electric circuits [16], chemical reactions [17–19], and population dynamics [18,20–25]. In the current paper, we use LV models to describe dynamics of optical soliton amplitudes. LV models have been widely used in the past to describe dynamics of population sizes [18,26,27] as well as the time evolution of chemical concentrations in certain chemical reactions [17–19,28,29]. The occurrence of supercritical Hopf bifurcations in LV models is of special interest, since in some cases, as the value of the bifurcation parameter is further changed, the limit cycle undergoes a period doubling cascade, and finally, chaotic dynamics is observed [19,21–24].

In the current paper, we provide the first demonstration of intermediate nonlinear dynamics of soliton amplitudes in multisequence soliton systems, described by the cubic NLS equation. For this purpose, we study propagation of multiple periodic soliton sequences in nonlinear optical waveguide coupler systems with weak linear gain-loss, weak broadband cubic gain-loss, and narrowband Kerr nonlinearity. The values of the gain-loss coefficients are chosen such that the equilibrium states of the LV models for amplitude dynamics undergo supercritical Hopf bifurcations. This enables observation of large stable oscillations of soliton amplitudes along ultra-long distances. The narrowband nature of the Kerr nonlinearity and the broadband nature of the cubic gain-loss lead to enhanced pulse pattern stability compared with the waveguides considered in Refs. [9–14]. Since two of the LV models that we study exhibit chaotic dynamics, our findings are an important step towards realization of spatio-temporal chaos with multiple sequences of colliding NLS solitons.

The rest of the paper is organized as follows. In Section 2, we present the coupled-NLS models for pulse propagation and the LV models for dynamics of soliton amplitudes. In Section 3, we present four examples for multisequence waveguide coupler systems, in which the soliton amplitudes exhibit large stable oscillations along ultra-long distances. For each of the four systems we present the predictions of the LV models, the results of numerical simulations with the coupled-NLS models, and a comparison. In Section 4, we carry out further investigation of the impact of Kerr nonlinearity and cubic gain-loss on pulse pattern stability and amplitude dynamics. Our conclusions are presented in Section 5.

2. Coupled-NLS and LV models

2.1. Coupled-NLS propagation models

We consider propagation of N sequences of optical pulses in an optical waveguide coupler, consisting of N close waveguides, where each sequence propagates through its own waveguide. We assume a multisequence setup, where the pulses in each sequence propagate with the same group velocity, but where the group velocity is different for pulses from different sequences [5–7]. Additionally, we assume that the sequences propagate in the presence of second-order dispersion, Kerr nonlinearity, and weak linear and cubic gain-loss. Thus, the propagation is described by the following system of N coupled cubic NLS equations [8,11,30]:

$$i\partial_z\psi_j + \partial_t^2\psi_j + 2|\psi_j|^2\psi_j = i\mathcal{F}^{-1}(G_j(\omega,z)\hat{\psi}_j)/2 - 2i\sum_{k=1}^{N}(1-\delta_{jk})\epsilon_{3jk}|\psi_k|^2\psi_j, \tag{1}$$