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Research paper

# Research on CDS pricing model with endogenous recovery rate

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ABSTRACT

Default probability and recovery rate are two important factors in credit risk management. These factors are negatively correlated with each other. In the previous literature, it is often assumed that these factors are independent or obey specified exogenous function. This simplified approach facilitates the calculation but may affect credit risk management. This paper deduces the functional relationship between the default probability and the expected recovery rate based on the Merton's structural model, calculates the expression of default probability using the Martingale method and the Laplace transform based on a reduced approach, and constructs a CDS pricing model with an endogenous recovery rate according to the non-arbitrage pricing principle.



## 1. Introduction

Credit risk has always been an important research object for financial researchers. However, researchers have focused on the default probability distribution before the financial crisis. There has been numerous insightful studies concerning the default probability distribution over the past 20 years; however, with the recovery rate (or the loss rate, they can be transformed into each other, as their sum is 1) has not garnered sufficient focus. In most of the literature, the recovery rate is assumed to be a fixed constant or a constant proportion of certain exogenous parameters. The serious consequences of the financial crisis have led to the recognition that the simple assumption of the recovery rate has seriously affected the accuracy of the pricing of credit derivatives and has affected the management of credit risk. Researchers began to focus on the recovery rate after the financial crisis. The Basel Committee revised the Basel Accord after the financial crisis. The three major elements of the credit risk of financial assets are modified as follows: default probability, default loss rate and default risk exposure.

In the literature, regarding credit risk management, the recovery rate is mainly assumed to be exogenous. Merton [1] notes that the default probability and the recovery rate should be determined by the market value of the company at maturity in the structural model, but he did not analyze how the market value affected the recovery rate. When Black and Cox [2] extends the Merton model to the first-passage time model, the recovery rate is assumed to be a constant proportion of outstanding debt value and to be independent with default probability. Gisecke [3] assumes that the recovery rate is a constant. The recovery rate is completely exogenous in the reduced-form model, via three means, as follows: (1) recovery of Treasury, Jarrow and Turnbull [4] argues creditor can be compensated by a fixed fraction of the value of

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equivalent non-defaultable bonds; (2) recovery of market value, Duffie and Singleton [5] assumed recovery rate is a fractional proportion of its market value at the instant before default; and (3) recovery of face value, Houweiling and Vorst [6] believe creditors should receive a fractional proportion of the face value of the bonds as the creditor's claim is part of the face value of the bond. These exogenous recovery rate assumptions simplify the calculation; however, they ignore the effect of recovery on the tail of the loss distribution and lead to potential model risks.

The literature about recovery rate mainly focus on two aspects, the modeling of the recovery rate distribution and the dependence between the recovery rate and the default probability:

The common method of modeling the recovery rate distribution is to assume that the recovery process obeys a specified stochastic process. Frye [7] uses Normal distribution; Pykhtin [8] investigates Logarithmic Normal distribution. Rosch and Scheule [9] use Logit-Normal distribution; Dullmann and Trapp [10] compare the Normal distribution, Logarithmic Normal distribution and Logit-Normal distribution. In all stochastic processes, Beta distribution is most popular for its flexible parameters and its simple estimation; Chen [11], Gupton and Stein [12], Bruche and Carlos [13], Chen et al. [14], J.P. Morgan's CreditMetrics, KMV's Portfolio Manager and Moody's Losscalc all use Beta distribution. To fit the "bimodal" in the empirical recovery rate, Ma et al. [15] use a two-factor Bernoulli process consisting of a constant and Beta distribution; Wang and Zhan [16] investigate a random process that is linearly combined by two Beta distributions. However, the empirical investigation shows that Beta distribution is very sensitive to parameters; thus, it cannot predict well [17]. In addition to the parametric approach, the research also investigates the non-parametric approach, such as ordinary least squares regression, fractional response regression, inverse Gaussian regression, regression tree and neural network et al; however, due to the lack of recovery rate data, the nonparametric methods are not accurate.

Previously, the recovery rate was assumed to be independent with default probability, nevertheless, if an asset pays a higher recovery rate given default, this means that it has an intuitively strong credit quality; thus, it will have a lower default probability, and vice versa. The empirical evidence also confirms this negative correlation between the recovery rate and the default probability; for example, Carty and Lieberman [18], Bakshi et al. [19], Hu and Perraudin [20], Altman et al. [21], Hamilton [22]. Specific to credit derivatives, Hull and White [23], and Das and Hanouna [24] found noteworthy negative dependence between the default probability and the recovery rate in CDS (Credit Default Swaps) and CDO (Collateralized Debt Obligation). The credit risk is underestimated if the randomness of the recovery rate is ignored.

Although the existence of the negative dependence between the default probability and the recovery rate is confirmed, the related research about the specific form and the internal mechanism of this dependence relationship is relatively minimal. Certain researchers assumed that the dependence relationship is a special function. For example, Bakshiet al. [25] and Charaf [26] assumed that the recovery rate is the exponential function of the density function of the default probability. Amraoui and Hitier [27] and Salah et al. [28] assumed that the recovery rate is a deterministic function of the copula factor Z of the Gaussian copula model. Certain researchers model this dependence relationship from the perspective of an asset intrinsic value process. Bruche and Carlos [29] introduced a 2-state credit cycle, in which there is a high default rate with a low recovery rate and a low default rate with a high recovery rate, respectively, which is driven by an unobserved Markov chain; however, the recovery rate in the model is a Beta distribution. Chava et al. [30] assumed that the default probability and the recovery rate depend on the same common factors. Hocht and Zagst [31] assumed that the short-rate, the dynamics of the market factor, the idiosyncratic risk factor, and the default intensity are provided by different Hull-White models, and the recovery rate is an exponential function of the market factor and the idiosyncratic factor. Boudreault et al. [32] model the dependence relationship using the firm's leverage ratio, the ratio of the liabilities process and the firm's assets process; the default density and the recovery rate are both a function of the leverage ratio. Although these models investigate the negative dependence between the default probability and the recovery rate from the internal value of the firm's assets, model risk remains for the given exogenous function between the recovery rate and the factors.

Modeling the default probability is an important factor for modeling the credit risk. This paper uses the reduced form approach in modeling the default probability for flexibility. Most of results assume the default intensity process obeys the geometric Brownian movement, however, there are many jumps in financial market data, while the underlying asset's path is continuous in geometric Brownian movement. It can be said that the accurate description of the jumps in financial market data has played a decisive role in pricing the financial asset; moreover, the model without jumps is not robust. Therefore, researchers introduced the Lévy process into financial asset pricing. Subordinators are nonnegative increasing Lévy process. They are very important ingredients for building Lévy-based models in finance. They can be used as time changes from other Lévy processes to pricing the assets, see for instance Luciano and Semeraro [33]; Stochastic differential equations driven by subordinators can be used to model the stochastic mortality process and the default intensity processes , see for instance Hainautand Devolder [34], Cariboni and Schoutens [35], Mai [36]; Subordinators can be used to modelling the default dependency in credit risk, see for instance Hering et al. [37], Balakrishna [38].

The structure model combines the default event with its repayment ability; it has intuitively strong explanatory power for the default event. Therefore, this paper uses the structure approach in modeling the recovery rate. Instead of being exogenous, given the relationship function between the default probability and the recovery rate, we calculate the endogenous relationship function between them from the firm's asset value process; therefore, we call it the "endogenous recovery rate". This paper uses the reduced form approach in modelling the default probability for flexibility, assumes the default intensity is a subordinator, calculates the closed form expression of default probability using the Martingale method and the Laplace transform, and constructs a CDS pricing model with endogenous recovery rate according to the non-arbitrage pricing principle.