Commun Nonlinear Sci Numer Simulat 61 (2018) 1–21

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Strong nonlinear dynamics of MEMS and NEMS structures based on semi-analytical approaches

Sarah Ben Sassi, Fehmi Najar*

Applied Mechanics and Systems Research Laboratory, Tunisia Polytechnic School, University of Carthage, B.P. 743, La Marsa, 2078, Tunisia

ARTICLE INFO

Article history:
Received 4 October 2016
Revised 22 January 2018
Accepted 30 January 2018

Keywords:
MEMS
NEMS
Reduced-Order model
Strong nonlinearity
Limit-cycle solution

ABSTRACT

The aim of this work is to derive a reduced order model (ROM) for an electrostatically actuated micro and nano structures which can be valid for relatively large displacements and high voltages. The study also presents a comparison between available semi-analytical ROMs based on Galerkin decomposition and Differential Quadrature Method (DQM) applied on micro and nano electrically actuated beams. The proposed ROM is an innovative approach to properly and accurately consider the electrostatic forces and account for the geometric nonlinearities in the derived equations of motion. The proposed ROM is derived by employing a mathematical model using the nonlinear Euler–Bernoulli theory, the DQM is employed to extract the mode shapes of the micro/nano beams. Discretized Galerkin approach in association with DQM is then utilized as a ROM and the Finite Difference Method (FDM) is employed to calculate limit-cycle solutions. The proposed ROM is used to investigate the effect of the strain's third order nonlinearities on the dynamic responses of micro/nano beams. A comparative study in terms of convergence and computational time is carried out on several ROMs based on modal decomposition for different mode shapes, and by performing or not the well known multiplication by the force denominator term in the equation of motion. As a benchmark, Local adaptive DQM (LaDQM) in combination with Long Time Integration are employed and their convergence is studied. Important outcomes of this study reveal the appropriateness of using only one mode in the discretized Galerkin approach for doubly clamped and cantilever at micro and nano scales, even when high voltages are applied. Besides, the results demonstrate that using high AC voltages for the actuation signal can lead to a non-negligible shifting effect regarding the equilibrium positions and the associated natural frequencies of a dynamic solution. An updated solution is proposed to predict this mismatch.

1. Introduction

Studying Micro/Nano Electro-Mechanical Systems (MEMS/NEMS) behavior requires knowledge not only in a multitude of engineering skills, such as electrical and mechanical engineering and material sciences, but also in the different interactions that can exist between them. For this reason, considerable efforts are being conducted by researchers to develop methodologies able to deal with challenges related to the modeling and design of such highly multi-physics and strongly nonlinear coupled systems. Basically, a M/NEMS device is composed by an elastic beam or plate subjected to deflect and oscillate

* Corresponding author.
E-mail address: fehmi.najar@ept.rnu.tn (F. Najar).

https://doi.org/10.1016/j.cnsns.2018.01.022
1007-5704/

under a combined DC and AC electric load whence emerges their role of acting or sensing on the neighboring environment. Due to their small scale and the acting electrostatic forces, the nature of Micro/Nano devices is characterized by a high nonlinear behavior. In fact, the dynamic behavior of the system is often described by partial-differential-integral equations that involve the effects of the electrostatic forces, large mechanical deformation, material nonlinearity, nonlinear damping and intermolecular forces.

Several accomplishments had been achieved by researchers in modeling and simulation of MEMS and NEMS devices during the last two decades. A number of the implemented methodologies are based on model-order reduction to handle the complex coupled mathematical problem. Despite the countless computational referenced methods, a constant dilemma persists till today between the growing demand for accuracy and efficiency of the implemented tools, on one hand, and the fastness of these computational methods on the other hand. In fact, depending on the accuracy of the solution or/and the computational costs, researchers had adopted different methodologies to study the nonlinear static and dynamic responses of MEMS and NEMS and investigate their stability.

Implemented Reduced Order Models (ROMs) are generally categorized according to nodes methods or domain methods [1]. The first category relies on decomposing the space domains into nodes then, evaluates the unknown variables of the problem as a function of these nodes while the second class of methods employs the modal projection to get ride of space dependency. Among nodes methods, much efforts in the past had been devoted to the use of Krylov subspace methods via employing either Arnoldi or lanczos methodologies [2]. This approach was shown to be computationally profitable but despite its widespread use, the method is still under revision and improvements [3].

Great attention was given to modeling MEMS devices through Differential Quadrature Method discretization (DQM). This method is known to be powerful and accurate by conserving nonlinear properties of the system in the developed ROM [4]. Kuang and Chen [5] and Najar et al. [6] were among first who developed discretized ROM using DQM to investigate the static and dynamic behavior of an electrically actuated doubly clamped micro actuators. Arani et al. [7] investigated the static pull in instability of piezoelectric nanoswitches using DQM and studied the influence of the air gap on the pull in voltage.

Assumed mode approach discretization remains the most common used ROM thanks to its simplicity and accuracy to express nonlinear dynamics of a structure. For MEMS, this method was used earlier by Younis et al. [8] to study the static and dynamic response of an electrically excited doubly clamped micro beam. Through their study, authors introduced an innovative procedure to handle with the complex electrostatic term resulting from Galerkin procedure. The methodology consists of multiplying the equation of motion by the force denominator term. Studying the efficiency of the proposed ROM, Younis et al. concluded through static and dynamic analysis that an acceptable convergence is reached with at least three symmetric modes of the micro beam and that perfect convergence was reached by using five symmetric modes. Nayfeh et al. [1] validated the proposed approach for large displacement using a shooting method.

Several followed contributions on MEMS/NEMS had approved this approach. Five modes in the Galerkin discretization method was adopted by Chaterjee and Pohit [9] to investigate the static and dynamic behavior of a micro free-clamped beam taking into account third order nonlinearities. Rasekh and Khadem [10] simulated the dynamic response of a cantilevered carbon nanotube using three terms in the modal decomposition and numerical integration. Nayfeh et al. [11] studied the dynamic pull-in of a doubly clamped micro beam actuated by DC and AC voltage. They used four modes in Galerkin approach combined with the shooting method to represent the dynamic response.

Tajaddodianfar et al. [12] opted for one-term modal projection and the MMS to study the chaotic motion of electrically actuated micro resonator. Qian et al. [13] studied the nonlinear vibrations of electrically actuated micro beam with fixed ends. They considered one term modal projection, multiplied the equation of motion by the force denominator term and applied the HAM.

This approach of multiplying the equation of motion by the force denominator term in order to decrease the problem complexity had gained a massive success among MEMS/NEMS community. Nevertheless, the huge amount of computational time caused by the high number of employed modes had pushed some researchers to seek for an alternative solution that reduces the number of retained modes, and hence the computational costs. For instance, Kacem et al. [14] expressed the electrostatic force applied on a nano cantilever beam using a Taylor series expansion up to the fifth order and used one mode in Galerkin decomposition to study the dynamic response of the structure.

A substitute to Taylor series approximation was suggested by Belardinelli et al. [15] through estimating the electrostatic force using Chebyshev polynomials up to third order term. Using this approach, authors reported good results when simulating the dynamic response of electrically excited doubly clamped micro beam using one mode-projection term and Runge–Kutta discretization. A simple fitting function of the force term was adopted by Amorim et al. [16] in order to reduce the retained mode shapes to a single term. Employing Range-Kutta method, they analyzed the chaotic regime of both micro and nano double fixed beams by including the effect of beam curvature and Casimir force.

Another alternative method proposed by Ouakad and Younis [17] consists of evaluating the integral term of a carbon nanotube electrostatic force (after performing Galerkin decomposition) using a trapezoidal rule. Applying a shooting method in conjunction with single mode shape approximation, they studied the dynamic response of a double clamped and cantilevered carbon nanotubes. One mode Galerkin decomposition was also used by Askari and Tahani [18] combined with four order Range–Kutta method to study the pull-in instability of a doubly clamped micro beam affected by mechanical shock and actuated by electric load. Here, the authors evaluated the electrostatic force integral term iteratively.