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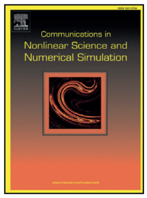

Research paper

Low-thrust displaced orbits by weak Hamiltonian-Structure-Preserving control

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ABSTRACT

For the requirements from some missions, the spacecraft is placed near the unstable region around the hyperbolic equilibrium. To stabilize the motion, a novel concept of the weak Hamiltonian-Structure-Preserving (HSP) control parameterized by the Coriolis acceleration is introduced. Based on the dynamical model of low-thrust displaced orbits, this paper searches for the mechanism in orbital stability change by the weak HSP control and provides specific strategies on how the control is implemented. Since the weak control has no effect on the topology of the original system, two invariant equilibria, a hyperbolic one and an elliptic one are solved. Dynamical system techniques are employed to investigate the controlled motions near the two equilibria, illustrating the Lyapunov orbit in different Coriolis acceleration cases and presenting two basic periodic modes measuring the bounded trajectories in the interior region. Controlled trajectories in the neck region are analyzed for their characteristics, classification and transition to make preparation for the stability change. One of the important contributions of this paper is to numerically demonstrate that the weak HSP control preserves the homoclinic orbits to the hyperbolic equilibrium as well as to the Lyapunov orbit, and another is to achieve the orbital transfer within, or even beyond KAM tori based on two methods in determining whether the controlled trajectory integrated from an initial point can exhibit transit.

1. Introduction

The concept of counter-acting gravity through a thrust vector was first proposed by Dusek in 1966 [1], who discovered that a spacecraft could be kept in a stable orbit around the collinear points through the use of continuous propulsive thrust. Large families of displaced orbits for a generic low-thrust propulsion were generated by considering the dynamics of the two-body problem in a rotating frame of reference [2,3]. The displaced orbits have been identified by solar sail or electronic thrusters, including three basic types of circular orbits by a thrust-induced acceleration [2–4], body-fixed hovering orbits by open-loop control [5], quasi-periodic displaced trajectories by a fixed thrust along the rotation axis of planet [6], a sequence of individual Keplerian arcs connected by slight impulse propulsion [7], elliptic displaced orbits with advanced thrust model [8], displaced geostationary orbit using hybrid sail propulsion [9], and so on. Such displaced non-Keplerian orbits (NKOs) have a diverse range of potential applications in both Sun-centered and planet-centered studies, such as the solar physics and an Earth synchronous orbit for continuous observations and space weather monitoring [10], displaced orbits above

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Nomenclature

ω	factor quantifying Coriolis acceleration
ρ	displaced orbit radius
z	displaced orbit height
h_z	angular momentum
κ	thrust acceleration
E	energy of the system
U	potential function
$\boldsymbol{n}$	thrust direction
$\boldsymbol{I}$	identity matrix
$\boldsymbol{J}$	symplectic matrix

Earth-Moon L_2 for lunar far-side communication [11], and a new family of NKOs displaced above or below the Earth's equatorial plane for the increasing number of available slots for geostationary communications satellites [12].

However, all of the mission orbits near hyperbolic equilibria are unstable, so that linear and nonlinear strategies have been investigated by a number of authors for the station-keeping requirements. McInnes [13] presented a passive control scheme of solar sail by designing the configuration to render the unstable subset of orbits linearly stable. Bookless [14] made analysis for the dynamics and control of the motions near the equilibrium, but his many conclusions are linear and local. Scheeres et al. [15] constructed a powerful tool Hamiltonian-Structure-Preserving (HSP) control that can stabilize the system in the sense of Lyapunov even in nonlinear situations. Bookless and McInnes [16] first developed the linear quadratic regulator technique in solar sail applications, which was then applied by Qian [17] to a nonlinear dynamical system of displaced orbits with a relatively favorable control precision.

Benefiting from the non-dissipative structure [18], the control of preserving Hamiltonian structure is preferable to be developed to stabilize the motions near the hyperbolic equilibrium. To extend the work of Scheeres, Xu and Xu [19] separated the HSP controller into two parts: the strong HSP characterized by the manifolds (stable, unstable, and center) and position feedback, and the weak HSP by the Coriolis acceleration and velocity feedback. They proved that a 2-degree-of-freedom Hamiltonian system can be stabilized by the strong HSP. Inspired by Xu and Xu [19], we focus on the investigation of the weak HSP control, where the first aim of our research is to investigate the influence of such station-keeping strategy on the dynamical behavior of motions near equilibria. Then, the second aim is to present how this control is implemented to stabilize the unstable motion.

In this paper, the displaced non-Keplerian orbits of low-thrust spacecraft are controlled using different Coriolis acceleration and velocity feedback. The developed control scheme is named as weak HSP control, for it presents the weak Hamiltonian-Structure-Preserving characteristics. Similarly, dynamical system techniques are used to analyze the nonlinear dynamics of this displaced orbit in a controlled system. Firstly, the weak controller is constructed and applied to a two-body problem, and two of the hyperbolic and elliptic equilibria which are independent of the control are solved. Then, the influence of the weak HSP control on the motions near the two equilibria is investigated by means of dynamical system techniques. The interior region is presented to be filled with the bounded trajectories, which can be measured by two basic periodic modes, referred to as the first type of periodic orbit (denoted as P.O.I) and the second type of periodic orbit (denoted as P.O.II). Subsequently, numerical simulations demonstrate that the stable and unstable manifolds in the interior region still connect homoclinically with each other. Next, the trajectories in the neck region are analyzed for their characteristics, classification and transition to make preparations for the stabilization of unstable orbits near hyperbolic equilibrium. Finally, the orbital transfer between different bounded orbits as well as the stable and unstable orbits is exemplified to show the function and application of the weak HSP control implemented.

2. Nonlinear dynamics of controlled displaced orbit

2.1. Hamiltonian dynamics of displaced orbit by low thrust

In general, displaced NKOs can be achieved by seeking equilibrium solutions to the two-body problem in a rotating frame of reference $\boldsymbol{R}$, as shown in Fig. 1, where the frame of reference $\boldsymbol{R}$ rotates with constant angular velocity $\boldsymbol{\omega}$ relative to an inertial frame $\boldsymbol{I}$. It is assumed that the spacecraft of mass m at position $\boldsymbol{r}$ has the active propulsion generating thrust $\boldsymbol{T}$ in direction $\boldsymbol{n}$, and the magnitude of the thrust-induced acceleration is constant.

This paper considers the two-body dynamics of a spacecraft by ignoring the higher harmonics of the gravitational potential and then generates NKOs displaced above the Earth using the low-thrust propulsion. To simplify, the system is nondimensionalized through a characteristic length Re, i.e., the Earth radius, and a characteristic time $T = \sqrt{Re^3/\mu}$, where μ is the gravitational parameter of the planet chosen to be unity, so that the nonlinear dynamical equations of motion for a spacecraft in the polar coordinates (ρ, z, θ) are given by [14]:

$$\ddot{\rho} = h_z^2/\rho^3 - \rho/r^3 \tag{1a}$$