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Research paper

Chaotic dynamics of two coaxially-nested cylindrical shells reinforced by two beams

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ABSTRACT

Non-linear dynamics and contact interactions of beam-shell structures composed of two closed cylindrical shells which are coaxially nested and reinforced by two beams located symmetrically on the shell external perimeter is studied. In the present work, clearances between the mentioned structural members are taken into account, two beams are subjected to harmonic loads, and the dissipation factors are neglected.

3D PDEs governing non-linear dynamics of beams and shells within the geometric theory of Novozhilov are employed, whereas the contact pressure is defined through Kantor's model. PDEs are reduced to ODEs by means of the FEM (finite element method), and the solution convergence is validated through different numbers of finite elements located along the structural members thickness and by employment of the Runge principle with respect to spatial coordinates. The Cauchy problem is solved by the explicit integration (Euler method), which allows one to carry out the computation without the need to define solutions in a few initial points.

Analysis of vibrations, including contact interactions, is realized with the use of methods of non-linear dynamics and the qualitative theory of differential equations, time histories/signals, phase portraits, Poincarè sections, Fourier spectra, wavelet-based analysis as well as the Lyapunov exponents.

1. Introduction

The studied complex slender structures in the form of cylindrical shells reinforced by beams on external shells sides are widely applied in many branches of nowadays technology, including the industry, aviation, rocket building, oil pipelines industry or chemical industry, in fabricating measurement devices, and, in particular in fabricating new devices which take into account the size-dependent structural behaviour.

There exists a huge number of references/papers/books devoted to modelling and analysis of constructions made of beams, plates and shells. Even if those structural members are modelled and studied independently, each member can

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be classified into one of numerous groups. For instance, in the case of beams, one may distinguish the console and clamped beams, curvilinear and straight beams, beams of circular or rectangular cross-section, as well as complex sandwich beams. There are also many different beam hypotheses/theories such as the Euler–Bernoulli [1], Kirchoff–Love [1], Timoshenko [2], Pelekh–Sheremetev, or Levinson–Reddy [3–6] ones. A comparison of different theories, with emphasis put on their advantages and disadvantages, has been carried out in reference [7]. In the case of plates and shells, one may investigate thick and thin plates, shells with different boundary conditions as well as triangular, rectangular, circle, ring, multi-layer, and cylindrical shells. This, as a rule, implies occurrence of a rich spectrum of the governing PDEs as well as the computational schemes yielding reliable and validating solutions.

Since the history of investigation of vibrations of beams and shells is long, only the recent tendency in development of modelling, numerical methods as well as methods of analysis of non-linear dynamics, optimization and control of the mentioned structural members are considered in the present work. The state-of- the-art of the literature devoted to studies on non-linear vibrations of shells up to year 2012 has been carried out in reference [8], whereas the more detailed overview of the state-of-the-art regarding thin shells from 2003 up to 2013 can be found in reference [9]. On the other hand, the state-of-the art aimed at an overview of the finite difference methods for solving linear and non-linear problems of deformation of the orthotropic and isotropic shells is presented in the work [10]. Reference [11] is devoted to an overview of static and dynamic problems of anisotropic and non-homogenous shells with variable parameters and their numerical solutions. Finally, the monograph of Amabili [12] presents a study on non-linear vibrations of plates and shells.

There is a vast number of industrial demands regarding exploitation of structural members such as beams, plates and shells considered separately and/or taking into account interactions between them, and there is a long list of published works aimed at detecting failures in elements of constructions with the help of the theory of non-linear dynamical systems [13–15]. The occurred failures/cracks locally change the flexibility of a structural element, which influences the eigenfrequency and vibration modes. The latter allows, with the help of a study of vibrations, to define a size of the failures and their localization. The cracks are modelled as simple massless components in the form of linear springs [16] and the rotational springs [17–19]. The multi-layer beams with one and few cracks are investigated in references [20,21].

A large class of investigations is devoted to multi-layer beams, plates and shells made from composite materials widely employed in nowadays industry. Free and excited vibrations of the multi-layer composite beams, plates and shells are studied with the use of different numerical methods (finite element method, finite difference method, and the Bubnov–Galerkin method in higher approximations [22,23] taking into account different types of non-linearity and boundary conditions). There are also works devoted to multi-layer mechanical systems [24–26] as well as publications aimed at analysing the synchronisation of vibrations of mechanical systems [27,28].

On the other hand, nano-beams, nano-plates and nano-shells are applied to produce micro-sensors and micro-actuators. This implies the recently developed directions of a study of the size-dependent beams, plates and shells [29–31]. The well-established theories and classical methods of solving problems devoted initially to vibrations of beams have been extended into investigations of functionally graded beams made from composite materials. In reference [32] the functionally graded beams have been analysed based on the Euler–Bernoulli theory. The multi-layer functionally graded beams have been analytically and experimentally studied in reference [33,34]. Free vibrations of those beams with an account of shear deformations of the first order have been investigated in the work [35]. In reference [36] an influence of temperature fields on the free and excited vibrations of the functionally graded beams is investigated. Dynamics of beams and plates as well as systems with infinitely many degrees of freedom have been attracting attention of many researchers [37,38].

In what follows, a state-of-the-art of the recent knowledge on non-linear dynamics of shells and beams is briefly described. One of important aspects in studying non-linear dynamics of structural members is the problem of investigating their chaotic vibrations. In the monograph [39] the 1D mathematical models of beams, panels of infinite length and shells with an account of geometric, physical, design and kinematic non-linearity have been considered. The formulated problems have been solved with a few qualitatively different methods, i.e. the finite difference method, the Bubnov–Galerkin method in higher approximations, and the Rayleigh–Ritz method in higher approximations. Novel and/or modified scenarios of transition from regular to timing chaotic and spatial-timing chaos have been detected and illustrated. Based on the analogy of universal features of chaos detected in simple mechanical systems, existence of a special universal transition into turbulence have been addressed in spatial problems of 1D mechanical structures. The employment of the wavelet-based analysis to study time-dependent frequency power spectrum of the Timoshenko and Pelekh–Sheremetev beams can be found in the works [39,40]. In reference [41] the mathematical model of non-linear dynamics of multi-layer beams with an account of rotational inertial effects have been derived and studied.

Free vibrations of a curvilinear thin beam are described in reference [42], whereas the spatial vibrations of a flexible spherical shell are analysed in [43]. Vibrations of closed cylindrical shells of variable thickness have been investigated in reference [44], whereas an influence of the orthotropy on the stress state of a goffer shallow cylinder has been studied by Grigorenko and Rozhok [45].

The separate and challenging subject is aimed at analysing the problem of contact interaction of beams, plates and shells. Kantor and Bohatyrenko [46] gave the theoretical basis for the contact problems. Chaotic synchronisations of plate-shell structures subjected to harmonic load are addressed in references [47,48].

In order to guarantee reliability of numerical results, it is highly required to carry out the laboratory experiments. Physical experiments devoted to the study of cylindrical shells have been described by Amabili and Alijani [49] and by Chen and Babcock [50].