Commun Nonlinear Sci Numer Simulat 62 (2018) 352–377

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Multistability of delayed complex-valued competitive neural networks with discontinuous non-monotonic piecewise nonlinear activation functions

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A R T I C L E   I N F O

Article history:
Received 24 April 2017
Revised 6 December 2017
Accepted 25 February 2018
Available online 27 February 2018

Keywords:
Multistability
Complex-valued delayed competitive neural networks
Discontinuous non-monotonic piecewise nonlinear activation function
Time-varying delays

A B S T R A C T

This paper is concerned with the problem of coexistence and dynamical behaviors of multiple equilibrium points for complex-valued competitive neural networks with discontinuous non-monotonic piecewise nonlinear activation functions. Without assuming the linearity or monotonicity of the activation functions, by virtue of the fixed point theorem and other analytical tools, several new sufficient conditions are developed to guarantee that the discontinuous complex-valued competitive neural networks have at least 16^n equilibrium points, among which 9^n are locally stable. In addition, some criteria for assuring the coexistence and local stability of multiple equilibria for real-valued competitive neural networks are established, which also show that the number of stable equilibria for the complex-valued neural networks is larger than the real-valued ones. A numerical simulation is conducted to illustrate the applicability and effectiveness of the obtained theoretical findings.

1. Introduction

In the past few decades, the neural network models considered in the dynamics analysis in most papers have only one single time scale, which implies that only the neuron activity is taken into account in these models. Namely, there is only one type of variables—the state variables of the neural neurons in these models. Nevertheless, in the dynamic neural network, since the learning process, the synaptic weights also change with time, and the change of the connection weights may have influence on the dynamics of the neural network. Hence, the competitive neural network with long-term memory and short-term memory has been widely concerned in theoretical research (see [1–5] and references therein). Generally speaking, competitive neural network contains two types of state variable: short-term memory (STM) and long-term memory (LTM). STM describes the rapidly changing behavior of neuronal dynamics, whereas LTM describes the slow behavior of unsupervised neuronal synapses. Until now, there have been considerable works on multistability of real-valued competitive neural networks [6–8].

As the extension of real-valued neural networks, the complex-valued neural network is one that processes information in the complex plane, namely, it has complex-valued input and output signals, complex-valued state variables, complex-valued connection weights and complex-valued activation functions. Due to more complicated properties than the real-valued neu-

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https://doi.org/10.1016/j.cnsns.2018.02.033
1007-5704/

ral network, it has been extensively studied [9–16]. However, so far, to the best of the authors' knowledge, multistability for complex-valued competitive neural networks (CVCNNs) has not been considered in the literature.

As we all know, the number of equilibrium points depends on the practical application of neural networks. Some applications, such as optimization, require that the network has only one equilibrium point of the mono-stable state. However, multiple stable equilibria of the neural networks are necessary whenever neural networks are used in the other applications such as image processing, associative memory and pattern recognition (see [17,18] and references therein). Multistability of neural networks has been extensively investigated [19–38]. In [19], by means of the fixed point theorem and other analytical tools, the authors developed some certain sufficient conditions that ensure that the n-dimensional discontinuous neural networks with time-varying delays can have at least 5^n equilibrium points, 3^n of which are locally stable and the others are unstable. In [22], by partitioning the state space, employing analysis approach and Cauchy convergence principle, sufficient conditions were established for the existence and local exponential stability of multiple equilibrium points, which ensure that 2^n-dimensional Cohen–Grossberg bidirectional associative memory neural networks with k-level discontinuous activation functions can have k^n equilibrium points. In [23], based on the geometrical properties of the discontinuous activation functions and the Brouwer's fixed point theory, the authors analyzed multistability for the complex-valued neural networks with discontinuous activation functions and time-varying delays and several sufficient criteria were obtained to assure the existence of 25^n equilibrium points. Among them, 9^n points are locally stable and $16^n - 9^n$ equilibrium points are unstable. In [29], the authors analyzed complex-valued neural networks with unbounded time-varying delays and some new sufficient conditions were derived to ensure that existence of $[(2\alpha+1)(2\beta+1)]^n(\alpha, \beta \geq 1)$ equilibrium points in which $[(\alpha+1)(\beta+1)]^n$ equilibrium points are locally μ-stable and the remaining equilibrium points $[(2\alpha+1)(2\beta+1)]^n - [(\alpha+1)(\beta+1)]^n$ are unstable for the considered neural networks. It must be pointed out that compared with the storage capacity of the real-valued neural networks, the storage capacity of the complex-valued ones with same dimensions is lager [34,35]. Consequently, it is meaningful to study the multistability of CVCNNs with discontinuous non-monotonic piecewise nonlinear activation functions.

It is well known that the multistability analysis of neural networks critically depends on the structures of activation functions chosen. So far, there exist two kinds of activation functions can be considered for neural networks, namely, the continuous activation function and the discontinuous activation function. There is no doubt that neural networks with discontinuous activation functions play an important role in dealing with dynamical systems possessing high-slope nonlinear elements. Thus, lots of significant works have been devoted to analyzing the dynamical behavior of neural networks with discontinuous activation functions (see [19–23] and references therein). Nevertheless, multistability for CVCNNs with discontinuous non-monotonic piecewise nonlinear activation functions has not been considered in the literature, which gives us another motivation to study the present research.

Generally speaking, in the hardware implementation of dynamical neural networks, the time delays inevitably occur in the neural networks on account of the finite switching speed of amplifiers and the transmission delays during the communication between neurons. Consequently, the research of the influence of time delay on the stability of the system has become a hot topic with great theoretical and practical importance [39–43].

Motivated by the above discussion, the objective of this paper is to establish some new sufficient conditions to guarantee that the CVCNNs with time-varying delays and discontinuous activation functions have at least 16^n equilibrium points, among which 9^n are locally stable. The remaining part of this paper is organized as follows. The CVCNNs with time-varying delays and discontinuous activation functions are presented and some preliminaries are briefly outlined in Section 2. In Section 3, some new sufficient conditions are derived towards the coexistence and explicit dynamical analysis of the multiple equilibrium points for the considered CVCNNs. Furthermore, some criteria are given towards the coexistence and local stability of multiple equilibria for the real-valued competitive neural networks. Section 4 provides one numerical example to demonstrate the feasibility and the effectiveness of the obtained results. Finally, some conclusions are drawn in Section 5.

Notation: The notations are quite standard. Throughout this paper, $\mathbb{R}$ and $\mathbb{C}$ show the set of real numbers and the set of complex numbers, respectively. $\mathbb{R}^n$ and $\mathbb{C}^n$ show, respectively, the n-dimensional Euclidean space and the n-dimensional unitary space. $\mathbb{R}^{n\times n}$ and $\mathbb{C}^{n\times n}$ are, respectively, the set of all $n \times n$ real matrices and the set of all $n \times n$ complex matrices. Let i be the imaginary unit, i.e., $i = \sqrt{-1}$. A^R and A^I denote, respectively, the real and the imaginary parts of matrix $A \in \mathbb{C}^{n\times n}$. $\overline{co}(M)$ is the closure of the convex hull for set M. For a vector-valued function $f(\cdot) = (f_1(\cdot), f_2(\cdot), \dots, f_n(\cdot))^T \in \mathbb{R}^n$, define $\overline{co}[f(\cdot)] = \overline{co}[f_1(\cdot)] \times \overline{co}[f_2(\cdot)] \times \dots \times \overline{co}[f_n(\cdot)]$, where $\overline{co}[f_k(\cdot)] = [f_k^-(\cdot), f_k^+(\cdot)]$, for $k = 1, 2, \dots, n$ and '$\times$' represents the Cartesian product. $\mathcal{B}([t_0 - \tau, t_0], \mathbb{R}^{4n})$ represents the Banach space of continuous vector-valued functions which map the internal $[t_0 - \tau, t_0]$ into $\mathbb{R}^{4n}$ with the topology of uniform convergence. For a vector $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n$, $\|x\|_\xi$ denotes that the norm of x with $\|x\|_\xi = \max_k \{\xi_k |x_k|\}$, where $\xi = (\xi_1, \xi_2, \dots, \xi_n)^T$ with $\xi_k > 0$ for $k = 1, 2, \dots, n$. 'D^-' denotes the upper left Dini derivative operator.

2. Neural network model and preliminaries

Consider a class of CVCNNs with time delays described by the following nonlinear delayed differential equations: