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Research paper

Chimera at the phase-flip transition of an ensemble of identical nonlinear oscillators

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ABSTRACT

A complex collective emerging behavior characterized by coexisting coherent and incoherent domains is termed as a chimera state. We bring out the existence of a new type of chimera in a nonlocally coupled ensemble of identical oscillators driven by a common dynamic environment. The latter facilitates the onset of phase-flip bifurcation/transitions among the coupled oscillators of the ensemble, while the nonlocal coupling induces a partial asynchronization among the out-of-phase synchronized oscillators at this onset. This leads to the manifestation of coexisting out-of-phase synchronized coherent domains interspersed by asynchronous incoherent domains elucidating the existence of a different type of chimera state. In addition to this, a rich variety of other collective behaviors such as clusters with phase-flip transition, conventional chimera, solitary state and complete synchronized state which have been reported using different coupling architectures are found to be induced by the employed couplings for appropriate coupling strengths. The robustness of the resulting dynamics is demonstrated in ensembles of two paradigmatic models, namely Rössler oscillators and Stuart-Landau oscillators.

1. Introduction

An ensemble of coupled oscillators is a veritable black box exhibiting a plethora of complex cooperative dynamical behaviors mimicking several real world phenomena [1–4]. Chimera state is such an intriguing emerging behavior that has been identified in an ensemble of identical oscillators with non-local coupling [5–15]. Since its identification, the notion of chimera has provoked a flurry of intense investigations because of the surprising fact that such an ensemble splits into two dynamically distinct domains, wherein all the oscillators evolve in synchrony in the coherent domain, while the oscillators in the incoherent domain evolve in asynchrony [16–18]. Earlier investigations on the phenomenon of chimera were restricted to nonlocal coupling, both in the weak and the strong coupling limits, giving rise to frequency [19,20] and amplitude chimeras [20–26], as it was believed that nonlocal coupling is a prerequisite for the onset of chimera in an ensemble of identical oscillators. Later investigations revealed the emergence of chimera states under global coupling [27–30], mean-field coupling [42] and even in nearest neighbor coupling [43,44]. In addition to the frequency and amplitude chimeras,

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other types of chimera states such as amplitude mediated chimeras [45], intensity induced chimeras [46], and chimera death [22,30,38,47,48] were also reported in the recent literature. Further, the robustness of chimera states was reported in [23,31], while chimera states and multi-cluster chimera states were also found in oscillators with time delayed feedback [32–34]. Very recently, different types of chimera states including imperfect chimera states and solitary states were also reported at the transition from incoherence to coherence [35,37].

Notably, chimera states have also been demonstrated in laboratory experiments. In particular, chimera states have been discovered in populations of coupled chemical oscillators, in electro-optical systems [49,50], electronic circuits [51] and in mechanical systems with two sub-populations of identical metronomes [52]. Surprisingly spatiotemporal patterns mimicking chimera states were also found in real world systems, which include the unihemispheric sleep of animals [53], the multiple time scales of sleep dynamics [54], and so on. Very recently, chimera states have also been reported in a network of two populations of Kuramoto model with inertia [55,56], which is also a model used in the analysis of power grids [57]. Further investigations on identifying the intricacies involved in the mechanism of the onset of chimera states is of vital importance from the perspective of neuroscience because of the concept of “bumps” of neuronal activity [58,60] associated with it.

In this paper, we report an interesting type of chimera arising out of the phase-flip bifurcation [66–69] of an ensemble of identical oscillators. The coherent domains of the chimera state are out-of-phase synchronized with each other, whereas the incoherent domain is constituted by the asynchronous oscillators. Both the out-of-phase synchronized coherent domains and the incoherent domain coexist simultaneously in an ensemble of identical oscillators for suitable parameter values attributing to the existence of a new type of interesting chimera state, namely chimera at the phase-flip bifurcation/transition of the ensemble of oscillators. In addition to the above, clusters with phase-flip transition (*PFC*), conventional chimera, solitary state and complete synchronized states are also found to exist from the completely incoherent oscillator ensemble as a function of the coupling strength. We find that the phase-flip transition among the oscillators in the ensemble is induced by a common environmental coupling [62], while the chimera state in the ensemble is induced by the nonlocal coupling among the identical oscillators. The phenomenon of phase-flip bifurcation/transition was shown to be induced by a common environmental coupling in two coupled oscillators in the recent past [62]. Abrupt transition from in-phase to anti phase synchronized oscillations among the oscillators as a function of a system parameter, where their relative phase difference changes from zero to π at the bifurcation/transition point, is known as phase-flip bifurcation/transition [66–68]. The indirect coupling arising from the common medium or from the environment of the dynamical systems was shown to be a source of several collective behaviors of real world systems (see Ref. [61–63] and references therein).

For instance, the indirect environmental coupling plays a crucial role in facilitating complex collective dynamics such as decoherence, co-ordinated rhythms in biological systems [60] and quorum sensing [64,65,73,74]. Our studies show that such a type of coupling along with a nonlocal coupling leads to a rich variety of complex collective behaviour like different types of chimeras, solitary state, and complete synchronized state reported in the literature using different coupling architectures. These are necessary and important complements to the current knowledge on the collective behaviors due to the dynamic environmental coupling. In addition, such a coupling facilitates a new type of interesting chimera, namely chimera at the phase-flip bifurcation/transition of the ensemble of oscillators for appropriate coupling strengths.

The plan of the paper is as follows. In Section 2, the emergence of chimera at the phase-flip transition and the conventional chimera from the nonlocally coupled Rössler oscillators with a common dynamic environment will be discussed. Similarly, in Section 3, we corroborate the generic nature of the results in the nonlocally coupled Stuart-Landau oscillators with a common dynamic environment. Finally, in Section 4, we provide the summary and conclusion. We present the existence of clusters with phase-flip transition, which is a special case of classical spatial chaos for different sets of initial conditions in Appendix A, the results of phase-flip transition between two coupled oscillators in Appendix B and also discuss the emergence of bistability among them in Appendix C.

2. Nonlocally coupled Rössler oscillators with common dynamic environment

We consider an ensemble of nonlocally coupled identical chaotic Rössler oscillators interacting through a common dynamic environment represented as

$$\dot{x}_i = -y_i - z_i + \frac{\varepsilon}{2P}\sum_{j=i-P}^{j=i+P}(x_j - x_i), \tag{1a}$$

$$\dot{y}_i = x_i + a y_i, \tag{1b}$$

$$\dot{z}_i = b + z_i(x_i - c) + k w_i, \tag{1c}$$

$$\dot{w}_i = -\alpha w_i + 0.5 z_i - \eta\left(w_i - \frac{q}{N}\sum_{j=1}^{N} w_j\right), \quad i = 1, 2, \ldots, N \tag{1d}$$