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Research paper

The detection of local irreversibility in time series based on segmentation

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ABSTRACT

We propose a strategy for the detection of local irreversibility in stationary time series based on multiple scale. The detection is beneficial to evaluate the displacement of irreversibility toward local skewness. By means of this method, we can availably discuss the local irreversible fluctuations of time series as the scale changes. The method was applied to simulated nonlinear signals generated by the ARFIMA process and logistic map to show how the irreversibility functions react to the increasing of the multiple scale. The method was applied also to series of financial markets i.e., American, Chinese and European markets. The local irreversibility for different markets demonstrate distinct characteristics. Simulations and real data support the need of exploring local irreversibility.

1. Introduction

One of the classical fields in econometrics is the quantitative analysis of financial time series [1,2] that has received over the last few decades valuable input from statistical physics, complex systems communities and nonlinear dynamics[3,4]. Particularly, the existence of long-range dependence, detected through multifractal measures has been used to quantify the level of development of a certain market [5]. Another related property about financial time series is that of time reversibility, i.e. the degree of dynamical invariance under time reversal. A stationary process $X(t)$ is said to be statistically time reversible (hereafter time reversible) if for every N, the series $\{X(t_1), \dots, X(t_N)\}$ and $\{X(t_N), \dots, X(t_1)\}$ have the same joint probability distributions [6–11]. This implies that a reversible time series and its time reversed are, statistically speaking, fairly probable. Reversible processes include the Gaussian linear processes, and are associated with processes at thermal equilibrium in statistical physics. Irreversibility of time series is an important issue in basic and applied science. Recently some work employs the concept of time series irreversibility to educe information about the entropy production of the physical mechanism generating the series, even though one ignores any detail of such mechanism [12,13], which is based on the relationship between statistical reversibility and physical dissipation [14,15]. In a more applicable content, it has been indicated that irreversibility of complex physiological series decreases with pathology or aging, being supreme in recent and healthy subjects [16–18]. The definition of time series reversibility is formal, so there is no a priori optimal algorithm to quantify it in practice. Recently, a few methods have been present [14–17,19,20] to explore time irreversibility. Most of them accomplish a time series symbolization, which typically make an empirical division of the data range [19] and subsequently analyze the symbolized series, by means of comparison of symbol strings occurrence in the forward and backwards series or using compression algorithms [13,21]. Then method of visibility graph is introduced. This is a time series analysis method

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which was proposed recently [22–26]. It utilizes graph theoretical concepts based on the mapping of a time series to a graph and the subsequent analysis of the related graph properties [27].

Some time series are not reversible entirely, while their local part may be reversible. Therefore, single scale irreversibility tests may lead to insufficient results especially when dealing with complex systems such as financial time series. Since some of the typical symbolic methods we know are local, the recently proposed multiscale algorithm based on coarse grained ideas can more extensively measure the irreversibility of the time series [12,17,28]. However, this method can not effectively reflect the local irreversible fluctuation of a time series. In this study we propose a multiple testing strategy for the detection of local irreversibility in time series, try to study the relationship between the irreversibility and the different parts of the sequence. It is profoundly explained that the multiscale irreversibility analysis based on segmentation not only allows us to explore the local irreversibility of the time series more deeply, but also can reveal the relationship between the whole and part of a time series when exploring progressively increasing time scale factor.

The remainder of the paper is organized as follows. Section 2 describes the methodology of multiscale irreversibility. And Section 3 presents the simulation of ARFIMA process and logistic map. Next, Section 4 outlines the database used and presents the results of irreversibility analyses for real stock markets to analyze the volatility scaling property. Finally, conclusive marks are given in Section 5.

2. Methodology

We proposed the new method to detect the local irreversibility property of a complex dynamical system such as stock market, so as to investigate the relationship between irreversibility and the level of localization.

The new procedure consists of five steps. The process is as follows:

Step 1: Let $X = (x(1), x(2), \dots, x(N))$ be a synchronous time series of length-N. We construct consecutive coarse-grained time series, $\{x_j^{(\tau)}\}$, determined by the scale factor τ. The coarse-graining process is like this: we first divide the original time series into nonoverlapping segments of length τ and then calculate the average of data points in each segment. Generally, each element of the coarse-grained time series $x_j^{(\tau)}$ are calculated referring to the equation

$$x_j^{(\tau)} = \frac{1}{\tau} \sum_{i=(j-1)\tau+1}^{j\tau} x_i, \quad 1 \le j \le N/\tau$$

For scale one ($\tau = 1$), the time series $\{x_j^{(1)}\}$ are the original time series. The length of each coarse-grained time series is equal to $N_d = N/\tau$.

Step 2: Let us continue processing the coarse-grained time series $\{x_j^{(\tau)}\}$. Determine the profile

$$X(i) = \sum_{j=1}^{i} (x_j^{\tau} - < x >), i = 1, \dots, N_d. \tag{1}$$

Step 3: Cut the profile $X(i)$ into $N_s \equiv [N_d/s]$ nonoverlapping segments of equal length s. Since the considered time scale s need not be a divisor of the record length N_s, a short section at the end of the profile will remain in most cases. In order not to ignore this part of the record, the same procedure is repeated starting from the other end of the record. Therefore, $2N_s$ segments are obtained altogether.

Step 4: Calculate the local trend for each segment v by a least-square fit of the data. Then, we define the detrended time series for segment duration s, denoted by $X_s(i)$, as the difference between the original time series and the fits

$$X_s(i) = X(i) - q_v(i), \tag{2}$$

where $q_v(i)$ is the fitting polynomial in vth the segment. The degree of polynomial can be varied in order to eliminate linear, quadratic or higher order trends in the fitting procedure. Different fitting orders will obtain different results. We frequently use larger than 1-order to analyze the time series, so as to get more accurate results. These methods distinguish in their capability of eliminating trends, for the detrending of the time series is done by the subtraction of the polynomial fits from the profile. In nth order, trends of order n in the profile and of order $n-1$ in the original series are eliminated. We calculate, for each of the $2N_s$ segments, the skewness:

$$F^3(v, s) = \gamma(X_s(i)) = \frac{< X_s(i) - < X_s(i) > >^3}{[\sigma^2(X_s(i))]^{3/2}}, \tag{3}$$

of the detrended time series $X_s(i)$ by averaging over all data points i in the vth segment, where

$$< X_s(i) > = \frac{1}{s} \sum_{i=1}^{s} X_s[(v-1)s + i], v = 1, 2, \dots, N_s,$$

$$< X_s(i) > = \frac{1}{s} \sum_{i=1}^{s} X_s[N - (v - N_s)s + i], v = N_s + 1, N_s + 2, \dots, 2N_s,$$