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Research paper

Lie symmetry analysis, exact solutions and conservation laws for the time fractional Caudrey–Dodd–Gibbon–Sawada–Kotera equation

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ABSTRACT

In this work, we investigate the Lie symmetry analysis, exact solutions and conservation laws (Cls) to the time fractional Caudrey-Dodd-Gibbon-Sawada-Kotera (CDGDK) equation with Riemann-Liouville (RL) derivative. The time fractional CDGDK is reduced to nonlinear ordinary differential equation (ODE) of fractional order. New exact traveling wave solutions for the time fractional CDGDK are obtained by fractional sub-equation method. In the reduced equation, the derivative is in Erdelyi-Kober (EK) sense. Ibragimov's nonlocal conservation method is applied to construct Cls for time fractional CDGDK.

1. Introduction

Fractional partial differential equations (FPDEs) appear in different field of science and engineering such as physics, biology, rheology, viscoelasticity, control theory, signal processing, systems identification and electrochemistry [1–8]. In order to describe nonlinear physical phenomena, obtaining exact solutions for nonlinear FPDEs is one of the most important aspects. This physical phenomenon may depend on both the time instant and the time history, which can be successfully modelled using the theory of derivatives and integrals of fractional order [1–8]. Recently, several methods have been applied to reach exact solutions of NLFPDEs in the literature. Among the techniques applied are the exp-function, fractional sub-equation, first integral , the G'/G-expansion, Lie symmetry method and many more [9–29].

Lie symmetry analysis is considered to be one of the effecient approaches for obtaining exact solutions of nonlinear partial differential equations (NLPDEs). In decades time, Lie's method has been described and applied in different textbooks and several physical and engineering models were analyzed [30–34].

However, application of the Lie symmetry analysis to FPDEs is quite new. Based on our observation, there are very small number of studies in the literature concerning FPDEs. For example, [15] analyzed the time fractional linear wave-diffusion equation and obtained a group of dilations. Using dilation symmetries invariant solutions were established. An attempt has

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been made to extend the Lie symmetry analysis to FPDEs such as invariant analysis [16–21]. Lie symmetry analysis, Cls and exact solutions of the seventh-order time fractional Sawada-Kotera-Ito equation was investigated in [22].

Furthermore, Cls are very important tools in the analysis and study of differential equations from mathematical and physical perspective [30]. One can find more details about Cls and their construction in [30,33,35–38,40]. The time FPDEs are classical PDEs where we replace its time derivative with fractional derivative. Here, we consider the time fractional CDGSK equation given by

$$u_t^{\alpha} + u_{xxxxx} + 30uu_{xxx} + 30u_x u_{xx} + 180u^2 u_x = 0. \tag{1}$$

We study symmetry reductions, exact traveling wave solutions and Cls for Eq. (1) by using Lie symmetry analysis, sub-equation method and Ibragimov's nonlocal conservation theorem [39], respectively. In Eq. (1), $\alpha(0 < \alpha \leq 1)$ is a parameter describing the order of the fractional time derivative. When $\alpha = 1$, Eq. (1) becomes

$$u_t + u_{xxxxx} + 30uu_{xxx} + 30u_x u_{xx} + 180u^2 u_x = 0. \tag{2}$$

The physical understanding of Eq. (2) was illustrated in [41]. Many authors obtained exact travelling wave solutions for Eq. (2) by different approaches [42–44].

The rest of the work is arranged in the following way: In Section 2, we give some preliminaries, Section 3, we give Lie symmetries and reduction for Eq. (1), exact travelling wave solutions for Eq. (1) are presented in Section 4, In Section 5, Cls for Eq. (1) are presented, we give a final remarks in Section 6.

2. Preliminaries

Consider RL fractional derivative [45,46] given by

$$D^{\alpha} f(t) = \begin{cases} \frac{d^n f}{dt^n}, & \alpha = n, \\ \frac{d}{dt^n} I^{n-\alpha} f(t), & 0 \leq n-1 < \alpha < n, \end{cases}$$

where $n \in \mathbb{N}$, $I^{\mu}f(t)$ is the RL fractional integral of order μ, and

$$I^{\mu} f(t) = \frac{1}{\Gamma(\mu)} \int_0^t (t-s)^{\mu-1} f(s) ds, \; \mu > 0$$
$$I^{\mu} f(t) = f(t)$$

and $\Gamma(z)$ is the Gamma function.

Definition 1. The RL fractional partial derivative is defined by

$$\partial_t^{\alpha} = \begin{cases} \frac{\partial^n f}{\partial t^n}, & \alpha = n, \\ \frac{1}{\Gamma(n-\alpha)} \frac{\partial^n}{\partial t^n} \int_0^t (t-s)^{n-\alpha-1} u(s,x) ds, & 0 \leq n-1 < \alpha < n. \end{cases} \tag{3}$$

If it exists, where ∂_t^n is the partial derivative of integer order n [45,46]. Fractional derivative has lots of definition [47–51].

2.1. Description of lie symmetry method for the time FPDEs

Herein, we present the description of Lie symmetry method for the time FPDEs [16–20].

Definition 2. The function $u = \theta(x,t)$ is an invariant solution of Eq. (4) associated with Eq. (7) such that

1. $u = \theta(x,t)$ satisfies Eq. (4).

2. $u = \theta(x,t)$ is an invariant surface of Eq. (5), this means that

$$\xi^2(x,t,\theta)\theta_t + \xi^1(x,t,\theta)\theta_x = \eta(x,t,\theta).$$

Consider time FPDEs [18,20] of the form

$$\partial_t^{\alpha} u = F(t, x, u, u_x, u_{xx}, u_{xxx}, u_{xxxx}, u_{xxxxx}), \tag{4}$$