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Research paper

Finite-time stability and synchronization of memristor-based fractional-order fuzzy cellular neural networks☆

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ABSTRACT

This paper mainly studies the finite-time stability and synchronization problems of memristor-based fractional-order fuzzy cellular neural network (MFFCNN). Firstly, we discuss the existence and uniqueness of the Filippov solution of the MFFCNN according to the Banach fixed point theorem and give a sufficient condition for the existence and uniqueness of the solution. Secondly, a sufficient condition to ensure the finite-time stability of the MFFCNN is obtained based on the definition of finite-time stability of the MFFCNN and Gronwall–Bellman inequality. Thirdly, by designing a simple linear feedback controller, the finite-time synchronization criterion for drive-response MFFCNN systems is derived according to the definition of finite-time synchronization. These sufficient conditions are easy to verify. Finally, two examples are given to show the effectiveness of the proposed results.

1. Introduction

In 1996, T Yang and L B Yang proposed the fuzzy cellular neural network (FCNN) for the first time and pointed out that it has a wide range of applications in image processing and fuzzy recognition [1,2]. Different from the traditional cellular neural network structure, the FCNN adds fuzzy logic (fuzzy AND and fuzzy OR) to its structure and maintains the local connection between cells. With the further study of the FCNN, some scholars have expanded its applications in more fields, such as association memory [3], optimization calculation [4], etc. It is well known that the stability is a prerequisite for ensuring the performance of FCNN when applied in these areas. Therefore, in the theoretical research on the FCNN, such as the existence, stability of solutions and synchronization have been widely concerned [1,5–13]. Li et al. investigated existence, uniqueness and the global asymptotic stability of fuzzy cellular neural networks with leakage delay, time-varying delays and continuously distributed delays, and some sufficient conditions are derived to ensure global asymptotic stability of the equilibrium point by using Lyapunov approach and the linear matrix inequality (LMI) method [8]. Bao et al. considered

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the existence, uniqueness and global robust exponential stability of the solution of interval fuzzy Cohen–Grossberg neural network with piecewise arguments, and new theoretical results of exponential stability are derived based on comparison principle [9]. Ratnavelu et al. studied the synchronization of fuzzy bidirectional associative memory neural network with mixed delays, and based on Lyapunov–Krasovskii functional and the LMI method, some sufficient conditions are obtained to ensure synchronization between master-slave systems [11]. We can see from these references that most of the research on stability and synchronization of fuzzy neural network is based on the Lyapunov stability theorem, but it is well known that the suitable Lyapunov function is not easy to be designed in most cases.

Due to the infinite memory property and hereditary property of fractional-order calculus, fractional-order systems have gained wide attention in the fields of mechanics, viscoelasticity, electrical circuits and engineering control [14–16]. Especially, Yang et al. made a lot of foundation work in the local fractional calculus, which made the theory of fractional calculus more perfect [17–19]. Recent studies show that fractional-order systems can provide a fundamental and general computation for single neurons, which helps to the information processing, stimulus anticipation and frequency-independent phase shifts of oscillatory neuronal firing [20]. The theory of fractional-order calculus has been a powerful tool in modeling and analyzing many scientific phenomena. Naturally, in order to make neural networks have better performance, many scholars incorporated the fractional calculus into neural networks to form a class of fractional-order neural network. Since Arena et al. firstly studied the dynamical characteristics of fractional-order cellular neural networks and found some interesting phenomena [21], fractional-order neural networks have attracted the considerable attention of scholars [22–30]. The memristor, which was first postulated by Chua in 1971 [31] and was successfully developed by the researchers at the Hewlett–Packard lab [32], its resistance value can vary with the current passing through it. The changing characteristics of the resistance is similar to the neuronal synapse when stimulated by the biological electrical signals. Meanwhile, because of its nanometer size, nonvolatile and pinched hysteresis characteristics, the memristor becomes the best component used to simulate the synapse so far. As a result, the researchers use the memristor instead of the resistors in traditional artificial neural network, thus forming memristor-based neural network. Because of the particularity of the fractional-order calculus and memristor, the dynamical behaviors such as stability and synchronization, which combine the memristor and the fractional-order neural network have been studied extensively [33–38]. However, the fuzzy neural network combined with the fractional order (abbreviated as the FFNN), or combined with the memristor (abbreviated as the MFNN), or simultaneously combined with the fractional order and memristor (abbreviated as the MFFCNN), is still in a initial stage. Liu et al. studied the robust adaptive lag synchronization of uncertain MFNN with time-varying delays, and obtained some sufficient conditions to ensure the lag synchronization of master-slave systems by utilizing Lyapunov function method and an adaptive controller [39]. Zhang et al. investigated the uniform stability problem in the sense of mean square of stochastic the FFNN with delay based on the stochastic analysis theory and the Banach fixed point principle [40]. Unfortunately, so far we have not found the relevant research results on the MFFCNN.

Stability analysis is one of the main research contents of neural network dynamics. Synchronization between complex networks has been widely used in many fields, such as secure communication, image processing, and so on. The synchronization of the drive-response systems can eventually be translated into the stability analysis of the error system. In addition, compared with the asymptotic stability or synchronization, the finite-time stability or synchronization of neural networks is paid more and more attention by researchers because of its practicality [34,35,41–45]. In the literature on the finite-time stability or synchronization in relation to the memristor, fractional order, or fuzzy neural networks, Bai et al. discussed the finite-time stability problem of discrete-time fuzzy Hopfield neural networks and proposed a sufficient condition by the concept of finite-time stability of fuzzy neural network, Lyapunov method and LMI [41]; The authors used similar method to study the finite-time synchronization problem of fuzzy cellular nerual networks with time-varying delays[42–44]; Zheng et al. investigated the finite-time stability and synchronization problem of memristor-based fractional-order Cohen–Grossberg neural network and derived some new sufficient conditions using set-valued map, differential inclusions, Gronwall inequality and a simple linear feedback controller [34]. But, there are few researches about the MFFCNN.

Inspired by the above discussion, considering that fractional-order calculus, memristor, and fuzzy logic may lead to more complex dynamic behaviors of neural networks, the main objective of this paper is to analysis the finite-time stability and synchronization problems of the MFFCNN in the sense of Filippov with the help of set-valued map, differential inclusions, Banach fixed point theorem, Gronwall–Bellman inequality and the definitions of finite-time stability and synchronization. The main contributions of this paper are summarized as follows.

(1) We study the finite-time stability and synchronization of the MFFCNN for the first time. By using the set-valued map and differential inclusions theory, the MFFCNN with discontinuous right-hand side is transformed into ordinary differential equations. Then we prove the existence and uniqueness of MFFCNN's solution under certain conditions by means of Banach fixed point theorem.

(2) We give the definitions of finite-time stability of the MFFCNN and synchronization of drive-response MFFCNN systems. According to these two definitions, some easy verifiable sufficient conditions for ensuring that the finite-time stability of the MFFCNN and the finite-time synchronization of drive-response the MFFCNN systems are obtained. Furthermore, our results can be easily extended to the memristor-based fractional-order neural networks without fuzzy logic.

(3) Two simulation examples are shown to illustrate the correctness of the main results.

The rest of this paper is organized as follows. we introduce the definition of the Caputo fractional calculus, the MFFCNN model, the definition of finite-time stability and synchronization and some lemmas in Section 2. In Section 3,